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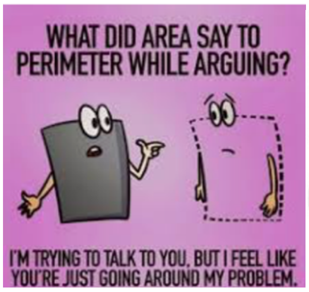
Area and Perimeter Of 2D Shapes Questions By Topic:



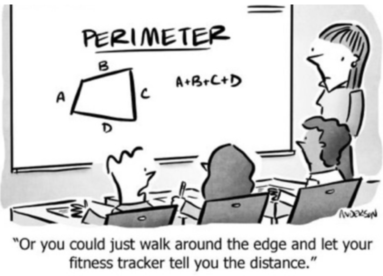


Table of Contents

all diagrams in this worksheet are not drawn to scale

1 Bronze

1.1 Counting Areas

1)

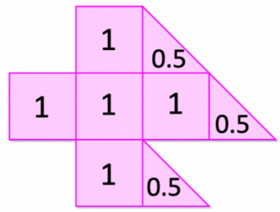
Way 1:

2 triangles= 1 square

$\Longrightarrow$5 squares + 3 triangles= 5 squares + 1.5 squares =6.5 squares

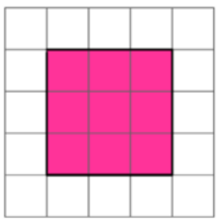
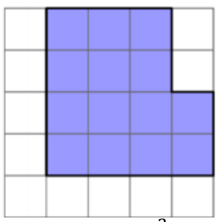
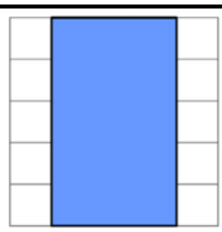
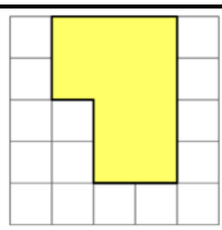
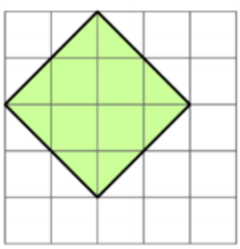
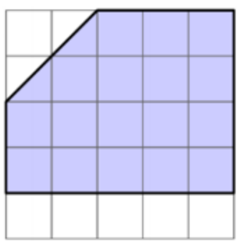
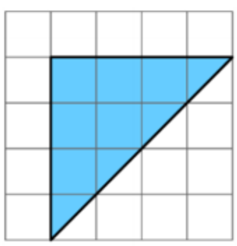
6.5(1)= $6.5cm^2$

Way 2:

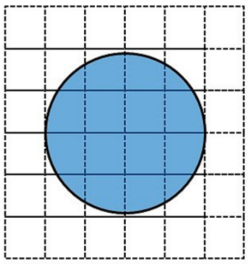


$$1+1+1+1+1+0.5+0.5+0.5 = 6.5\ cm^2$$

2)

 $6\ cm^2$	 $14\ cm^2$	 $15\ cm^2$	 $10\ cm^2$
 2 triangles= 1 square We have 4 squares + 8 triangles = 4 squares + 4 squares = 8 squares $8\ cm^2$	 2 triangles= 1 square We have 17 squares + 2 triangles = 17 squares + 1 square = 18 squares $18\ cm^2$	 2 triangles= 1 square We have 6 squares + 4 triangles = 6 squares + 2 squares = 8 squares $8\ cm^2$	

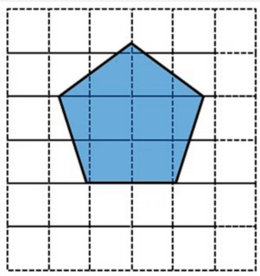
3)



Fully filed counts as 1
More than half a square counts as 1
Less than half a square counts are 0
Exactly half a square counts as 0.5

Here we have:
4 fully filled (4×1)
8 more than half squares (8×1)
4 less than half squares (4×0)

$12\ cm^2$

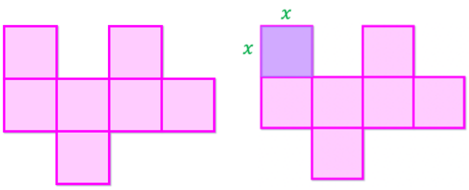


Fully filed counts as 1
More than half a square counts as 1
Less than half a square counts are 0
Exactly half a square counts as 0.5

Here we have:
4 fully filled (4×1)
4 more than half squares (4×1)
6 less than half squares (6×0)

$8\ cm^2$

4)



There are 7 squares

Areas of 1 square$=\frac{63}{7} = 9$

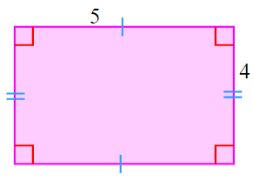
$x =$ length of one square$= \sqrt{area} = \sqrt{9} = 3\ cm$

The shape is made up of 16 lengths

Perimeter$= 3(16) = 48\ cm$

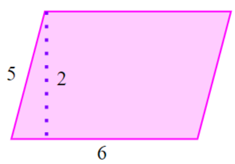
1.2 Simple Shapes

5)



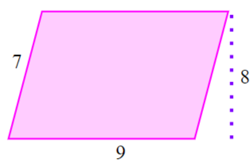
Perimeter = $5 + 4 + 5 + 4 = 18$

Area=length×width$= 5(4) = 20$



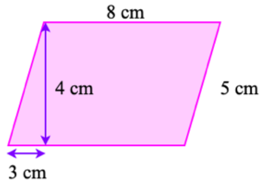
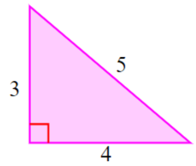
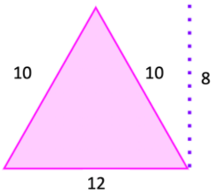
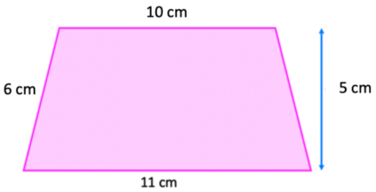
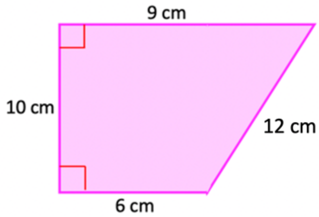
Perimeter$=5 + 5 + 6 + 6 = 22$

Area=base×height$= 6(2) = 12$



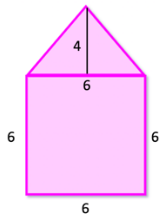
Perimeter$=7+7 + 9 + 9 = 32$

Area=base×height$= 9(8) = 72$

 Perimeter=$5+5+8+8=26\ cm$ Area=base×height= $8(4)=32\ cm^2$	 Perimeter=$5+4+3=12$ Area=$\frac{1}{2}\times$ base $\times$ height $=\frac{1}{2}(3)(4)=6$	 Perimeter=$10+10+12=32$ Area $=\frac{1}{2}\times$ base $\times$ height $=\frac{1}{2}(12)(10)=60$
 Perimeter=$11+10+6+6=33=cm$ Area= $\frac{1}{2}$(sum of parallel sides) $\times$ height $=\frac{1}{2}(10+11)(5)=52.5\ cm^2$	 Perimeter= $9+6+10+12=37\ cm$ Area= $\frac{1}{2}$(sum of parallel sides) $\times$ height $=\frac{1}{2}(9+6)(10)=75\ cm^2$	

1.3 Compound Shapes

6)

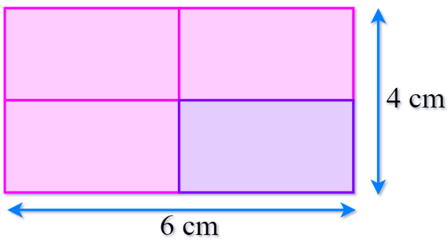


Area of the square= $6(6)=36$

Area of the triangle $\frac{6(4)}{2}=12$

$36+12=48\ cm^2$

7)

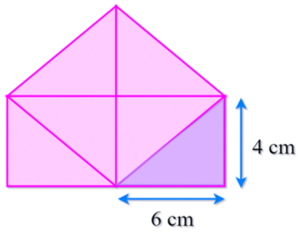


Area of entire flag= $6(4)$

Area of purple rectangle $=\frac{6(4)}{4}=\frac{24}{4}=6\ cm^2$

8)

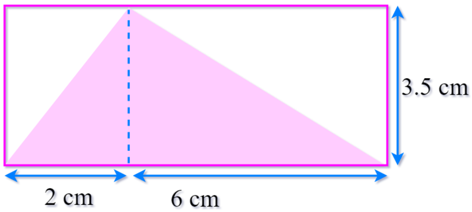
This shape is made up of 6 identical triangles



Area of 1 triangle= $\frac{1}{2}(6)(4) = 12$

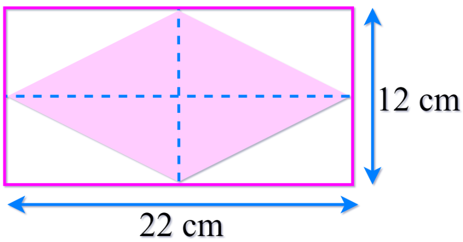
Area of 6 triangles= $6(12) = 72\ cm^2$

9)



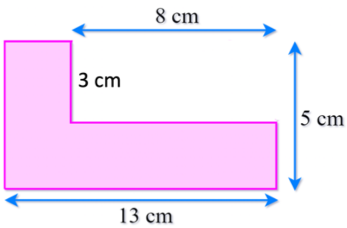
Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}(8)(3.5) = 14\ cm^2$

10)



Way 1: Area of 2 triangles	Way 2: Area of kite
$2\left[\frac{1}{2} \times 22 \times 6 = 132\right]$ Or $2\left[\frac{1}{2} \times 12 \times 11 = 132\right]$ $= 132\ cm^2$	Area of a Kite= $\frac{1}{2}(\text{product of the diagonals})$ $= \frac{1}{2}(22 \times 12) = 132\ cm^2$

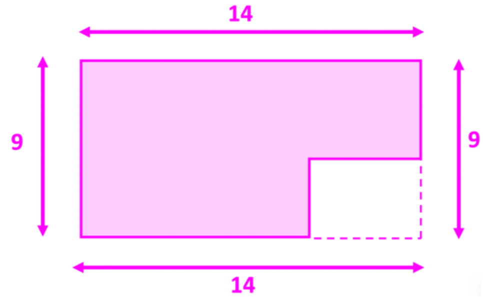
11)



The key to solving this question is to use the fact that opposite sides have the same lengths

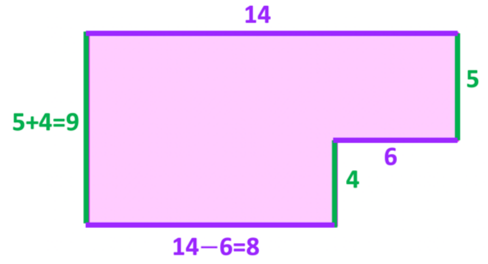
Perimeter

Way 1: Make into a complete rectangle



$14 + 14 + 9 + 9 = 46\ cm$

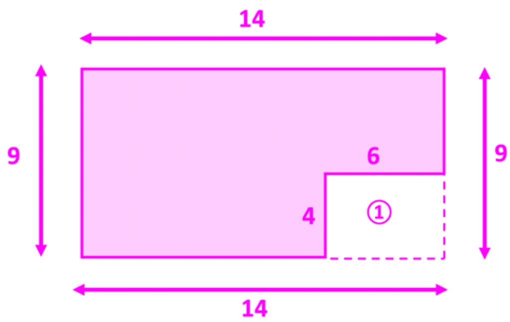
Way 2: Break each individual side down



$14 + 8 + 6 + 9 + 4 + 5 = 46\ cm$

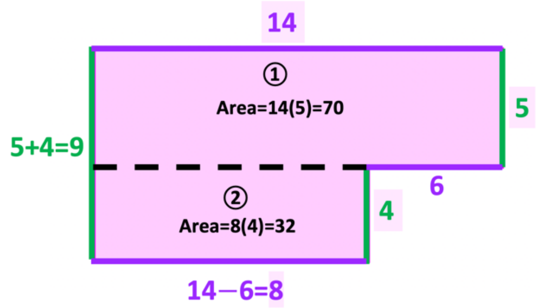
Area

Way 1: Make into a complete rectangle



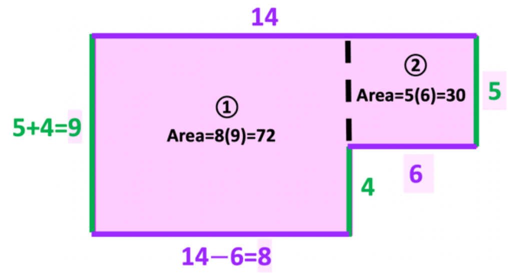
$14(9) - 6(4) = 102\ cm^2$

Way 2: Break each individual side down

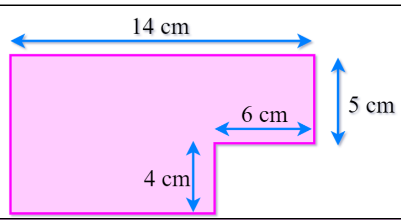


$32 + 70 = 102$

Or we can split up like this:

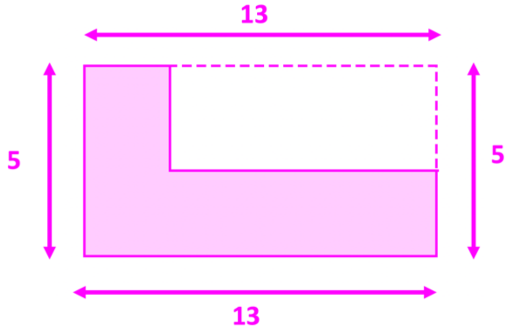


$72 + 30 = 102\ cm^2$

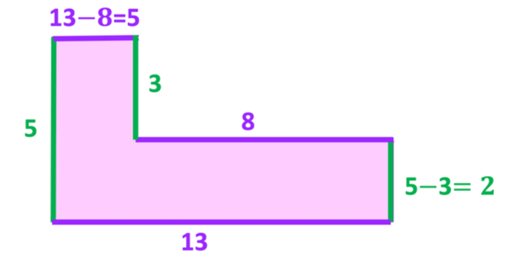


Perimeter

Way 1: Make into a complete rectangle	Way 2: Break each individual side down



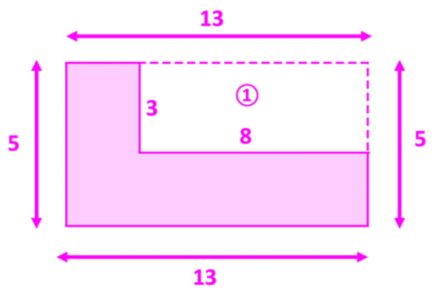
$13 + 13 + 5 + 5 = 36\ cm$



$5 + 8 + 13 + 5 + 3 + 2 = 36\ cm$

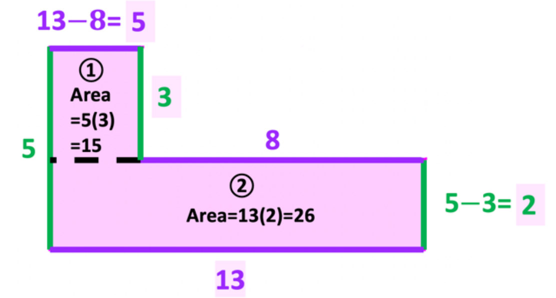
Area

Way 1: Make into a complete rectangle



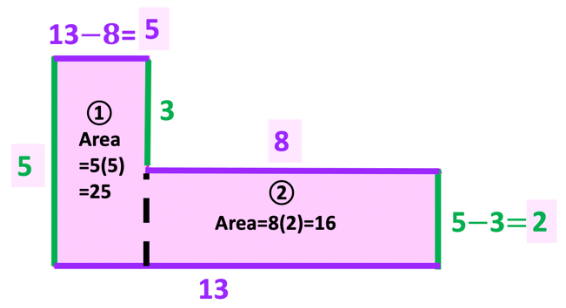
$13(5) - 3(8) = 41\ cm^2$

Way 2: Break each individual side down

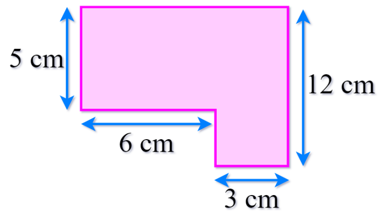


$15 + 26 = 41\ cm^2$

Or we can split up like this:

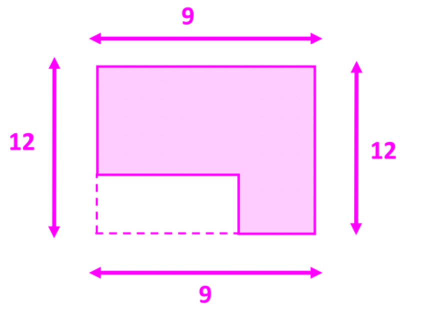


$25 + 16 = 41\ cm^2$

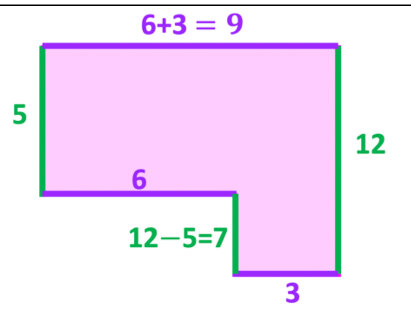


Perimeter

Way 1: Make into a complete rectangle	Way 2: Break each individual side down



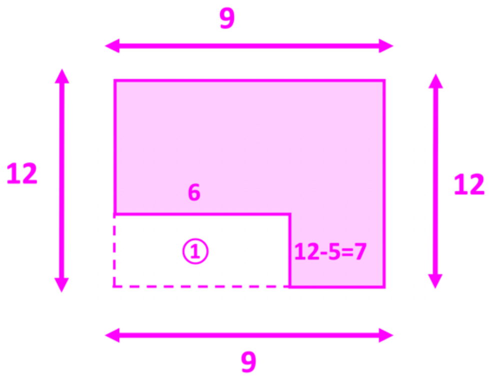
$9 + 9 + 12 + 12 = 42\ cm$



$9 + 6 + 3 + 5 + 12 + 7 = 42\ cm$

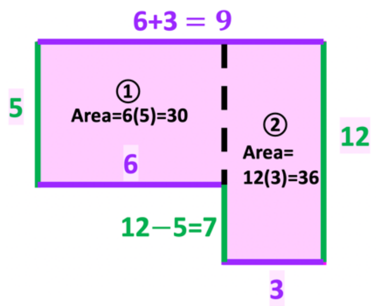
Area

Way 1: Make into a complete rectangle

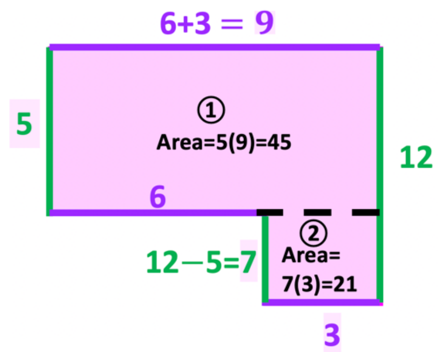


$12(9) - 6(7) = 66\ cm^2$

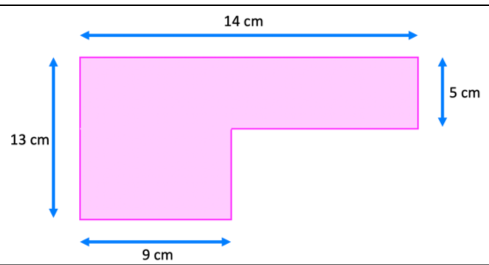
Way 2: Break each individual side down



$30 + 36 = 66\ cm^2$

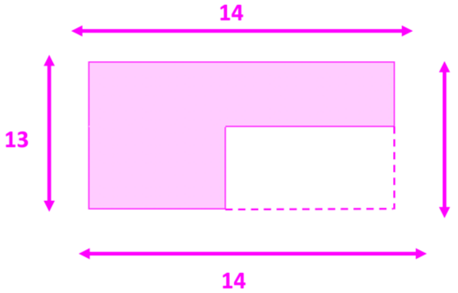


$45 + 21 = 66\ cm^2$

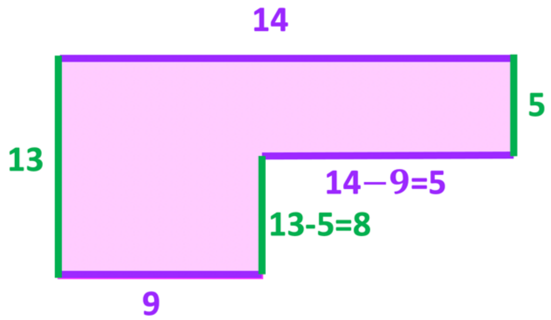


Perimeter

Way 1: Make into a complete rectangle	Way 2: Break each individual side down



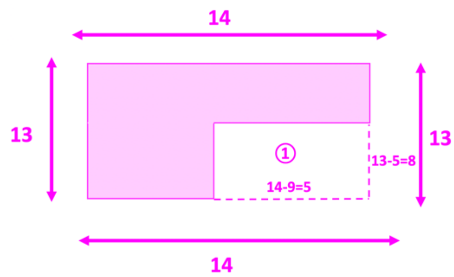
$13 + 13 + 14 + 14 = 54\ cm$



$14 + 9 + 5 + 13 + 8 + 5 = 54\ cm$

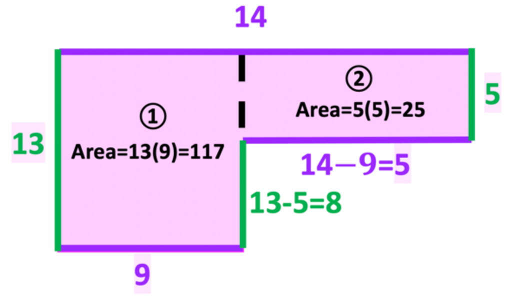
Area

Way 1: Make into a complete rectangle

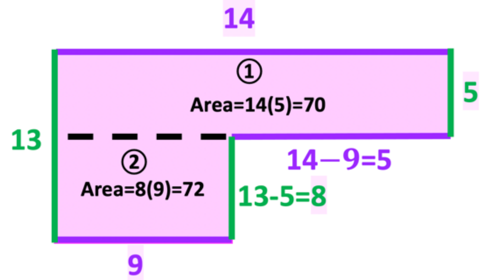


$13(14) - 8(5) = 142\ cm^2$

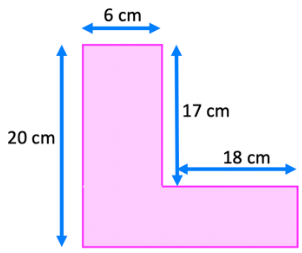
Way 2: Break each individual side down



$117 + 25 = 142\ cm^2$

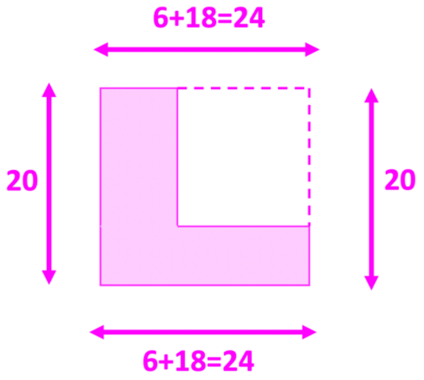


$70 + 72 = 142\ cm^2$

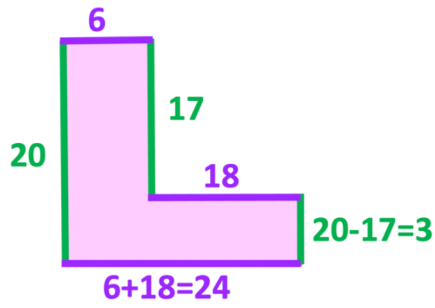


Perimeter

Way 1: Make into a complete rectangle	Way 2: Break each individual side down



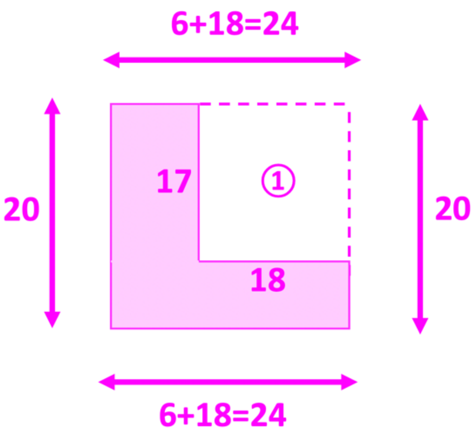
$24 + 24 + 20 + 20 = 88\ cm$



$6 + 18 + 24 + 20 + 17 + 3 = 88\ cm$

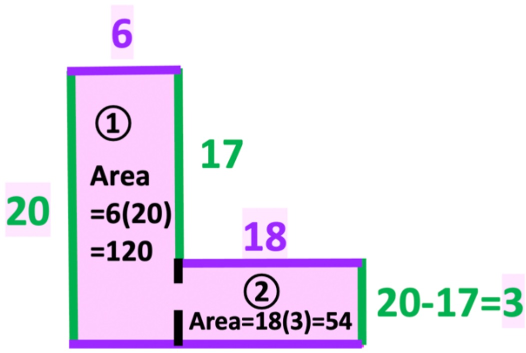
Area

Way 1: Make into a complete rectangle

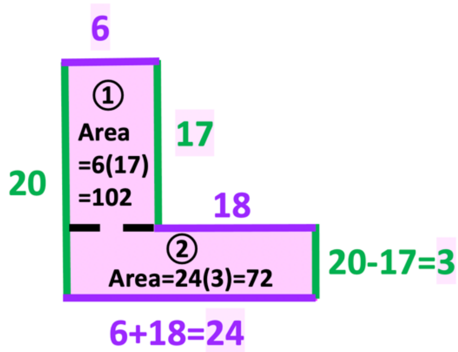


$20(24) - 17(18) = 174\ cm^2$

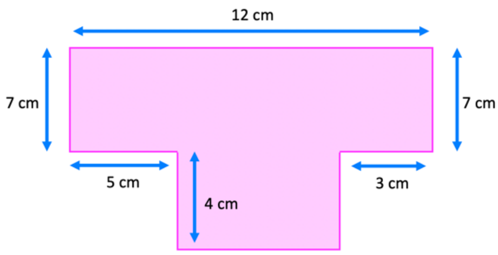
Way 2: Break each individual side down



$120 + 54 = 174\ cm^2$

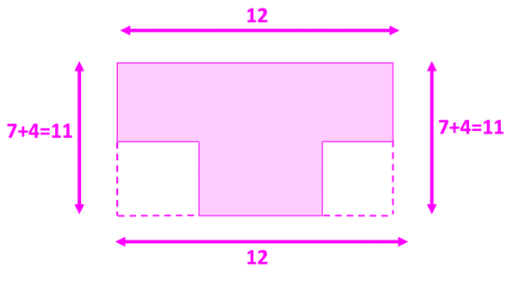


$102 + 72 = 174\ cm^2$

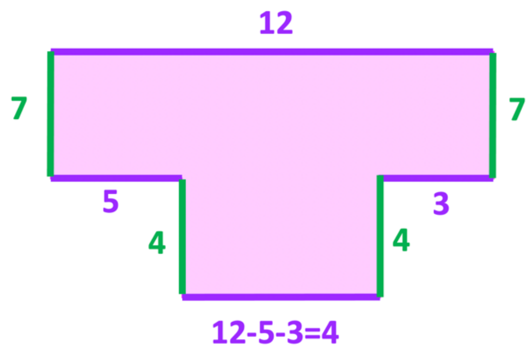


Perimeter

Way 1: Make into a complete rectangle	Way 2: Break each individual side down



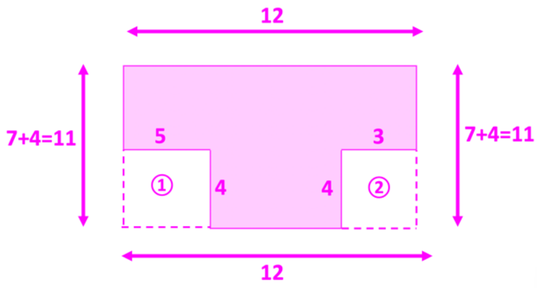
$12 + 12 + 11 + 11 = 46\ cm$



$12 + 5 + 3 + 4 + 7 + 7 + 4 + 4 = 46\ cm$

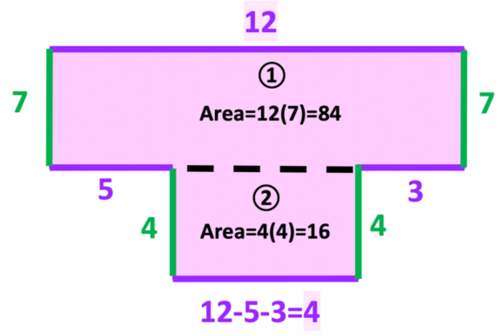
Area

Way 1: Make into a complete rectangle

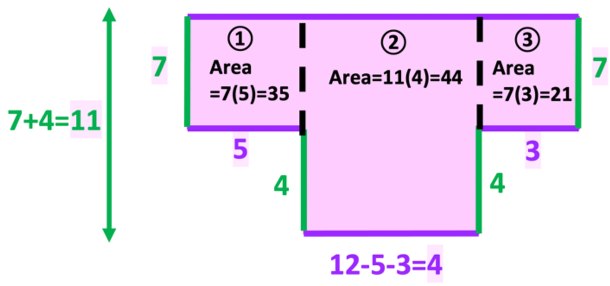


$12(11) - 5(4) - 4(3) = 100\ cm^2$

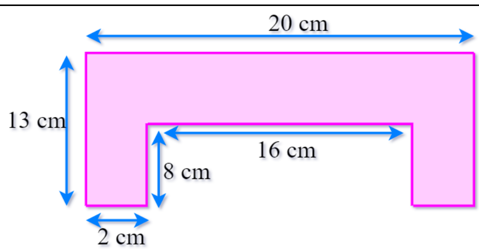
Way 2: Break each individual side down



$84 + 16 = 100\ cm^2$

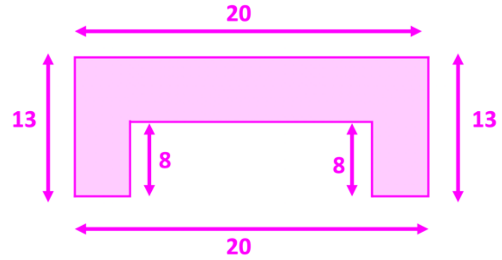


$35 + 44 + 21 = 100\ cm^2$

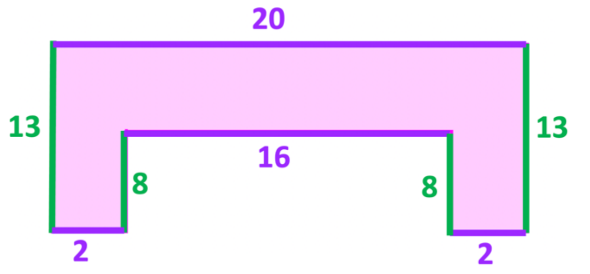


Perimeter

Way 1: Make into a complete rectangle	Way 2: Break each individual side down



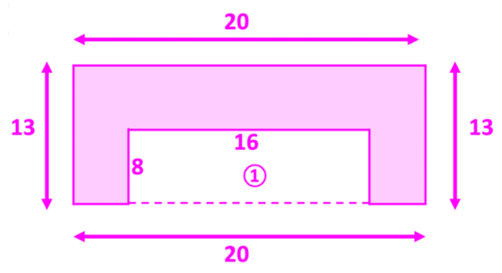
$20 + 20 + 13 + 13 + 8 + 8 = 82\ cm$



$20 + 16 + 2 + 2 + 13 + 13 + 8 + 8 = 82$ cm

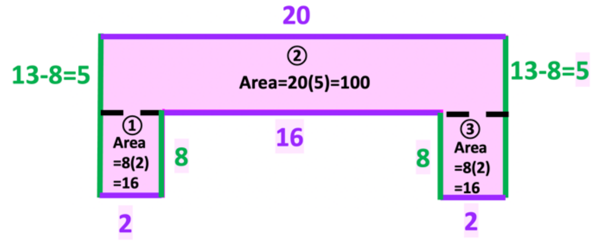
Area

Way 1: Make into a complete rectangle

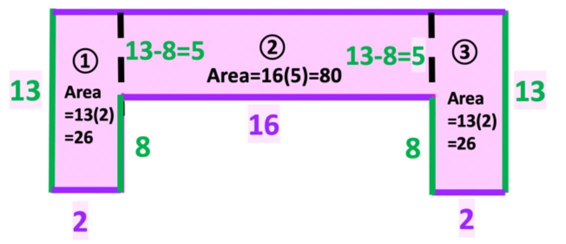


$20(13) - 8(16) = 132\ cm^2$

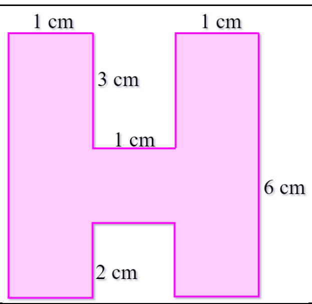
Way 2: Break each individual side down



$16 + 100 + 16 = 132\ cm^2$



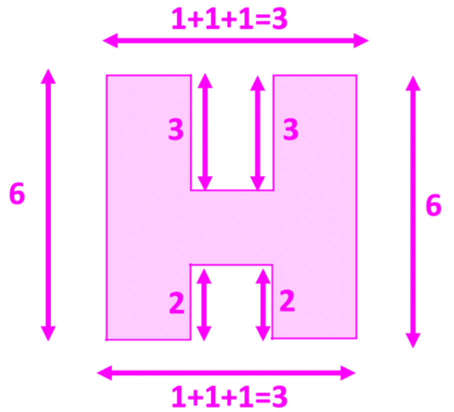
$26 + 80 + 26 = 132\ cm^2$



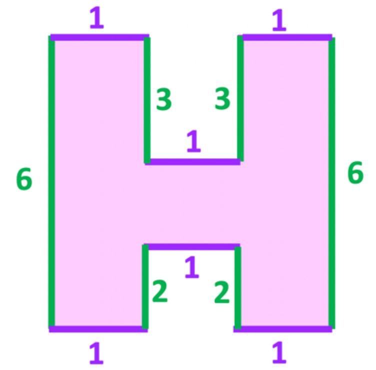
Perimeter

Way 1: Make into a complete rectangle

Way 2: Break each individual side down



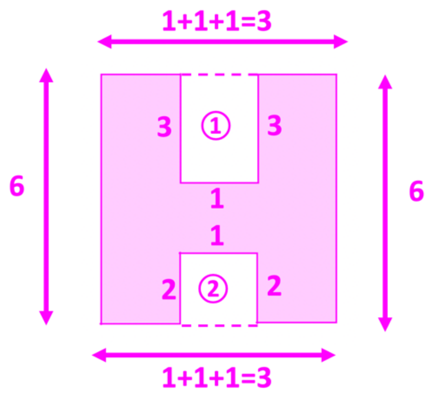
$6+6+3+3+3+3+2+2=28\ cm$



$6+6+3+3+2+2+1+1+1+1+1+1=28\ cm$

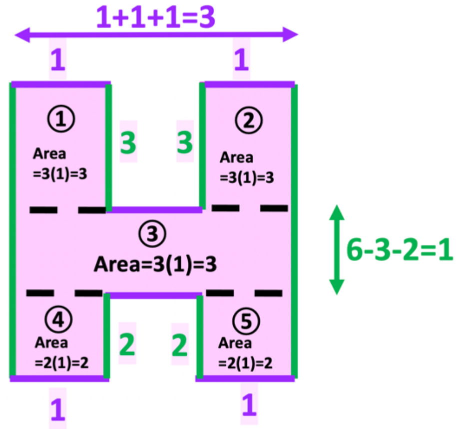
Area

Way 1: Make into a complete rectangle

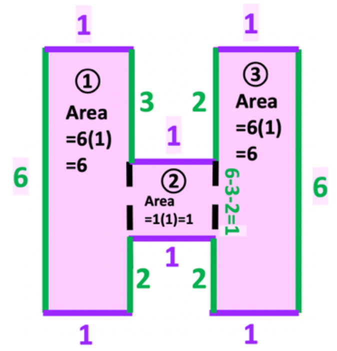


$6(3)-1(2)-1(3)=13\ cm^2$

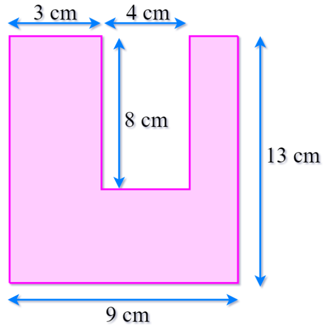
Way 2: Break each individual side down



$3+3+3+2+2=13\ cm^2$

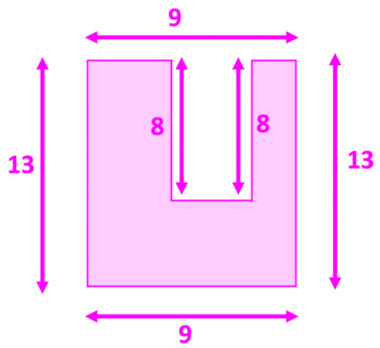


$6+1+6=13\ cm^2$



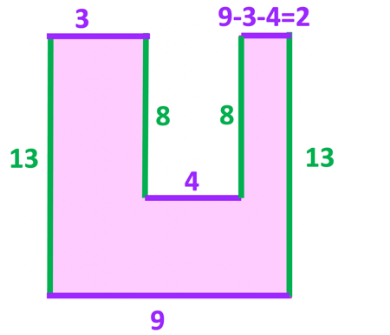
Perimeter

Way 1: Make into a complete rectangle



$13 + 13 + 9 + 9 + 8 + 8 = 60\ cm$

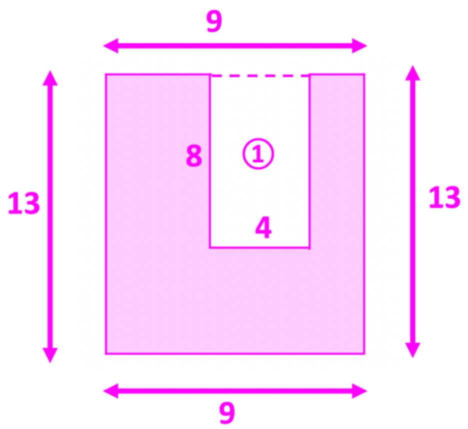
Way 2: Break each individual side down



$13 + 13 + 8 + 8 + 3 + 2 + 4 + 9 = 60\ cm$

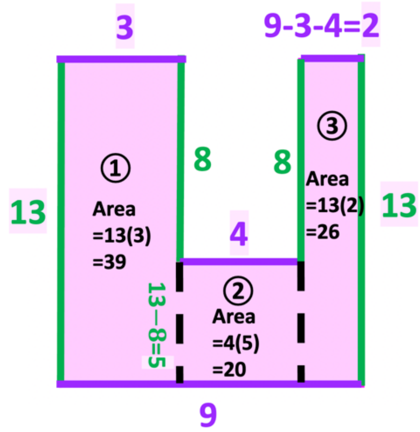
Area

Way 1: Make into a complete rectangle

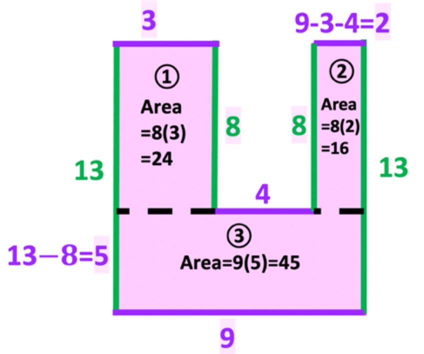


$13(9) - 8(4) = 85\ cm^2$

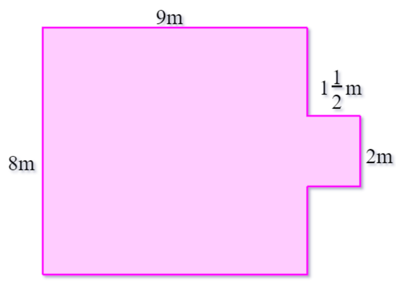
Way 2: Break each individual side down



$39 + 20 + 26 = 85\ cm^2$

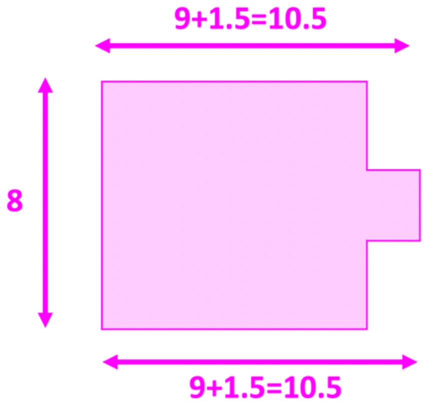


$24 + 16 + 45 = 85\ cm^2$

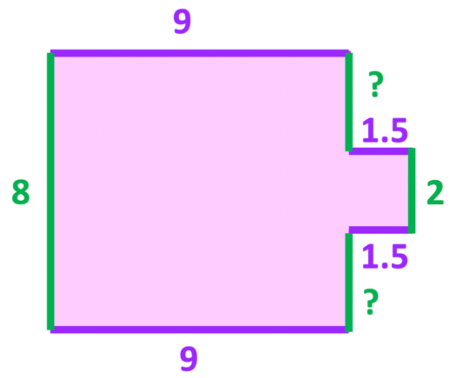


Perimeter

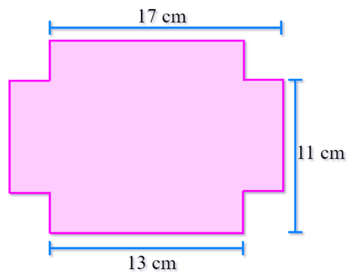
Way 1: Make into a complete rectangle	Way 2: Break each individual side down



$10.5 + 10.5 + 8 + 8 = 37\ cm$

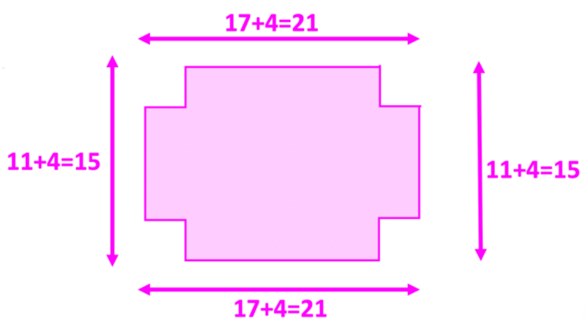


We can't do it this way as we don't know the ? lengths cm



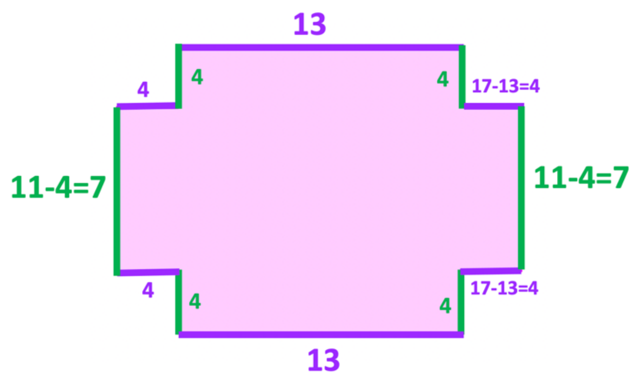
Perimeter

Way 1: Make into a complete rectangle



$21 + 21 + 15 + 15 = 72\ cm$

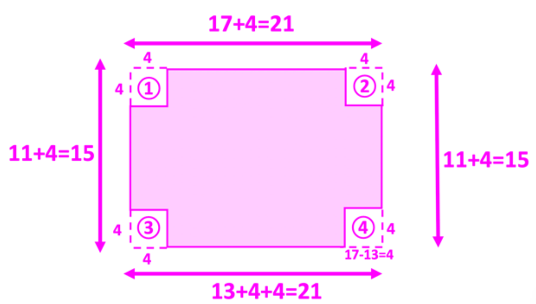
Way 2: Break each individual side down



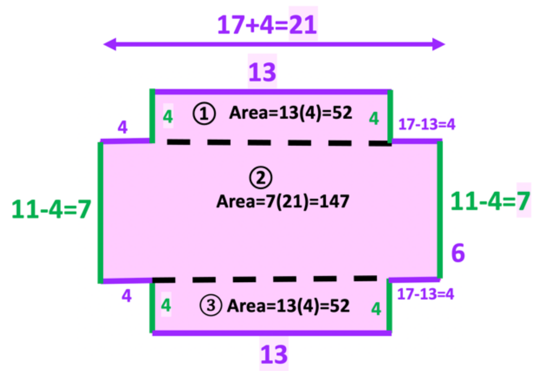
$13 + 13 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 7 + 7 = 72\ cm$

Area

Way 1: Make into a complete rectangle

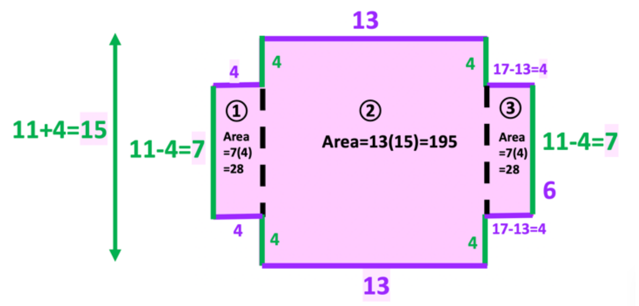


Way 2: Break each individual side down



$15(21) - 4(4) - 4(4) - 4(4) - 4(4) = 251\ cm^2$

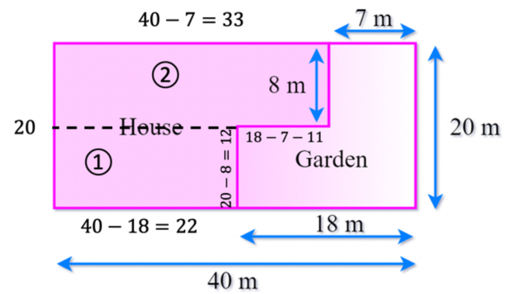
$52 + 147 + 52 = 251\ cm^2$



$28 + 195 + 28 = 251\ cm^2$

12)

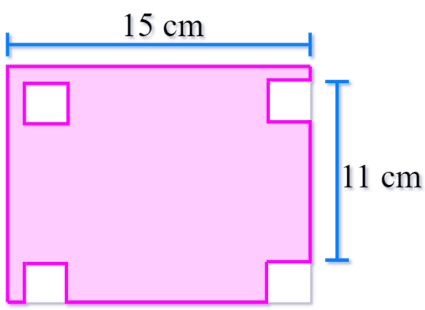
There are many ways to split and do this question. Let's choose the following way:



Area of house: Area of ① + Area of ② $= 22(12) + 8(33) = 528\ m^2$

Area of garden = area of whole rectangle – area of house $= 40(20) - 528 = 272\ m^2$

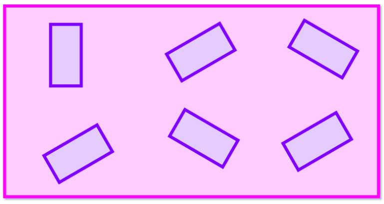
13)



Area of entire rectangle – Area of 4 squares

$= (13.5)(15) - 4(2.5)(2.5) = 177.5\ cm^2$

14)



Area of entire rectangle – Area of 6 smaller rectangles

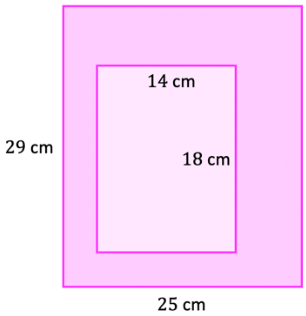
$= 37(18) - 6(2)(3.5) = 624\ cm^2$

15)

Area of larger rectangle – Area of smaller rectangle

$= 15(12) - 9(6) = 126\ cm^2$

16)

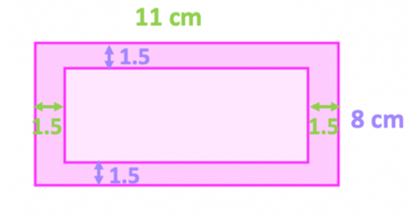


Area of larger rectangle $= 29(25) = 725\ cm^2$

Area of smaller rectangle $= 14(18) = 252\ cm^2$

Area of darker pink region $= 725 - 252 = 473\ cm^2$

17)



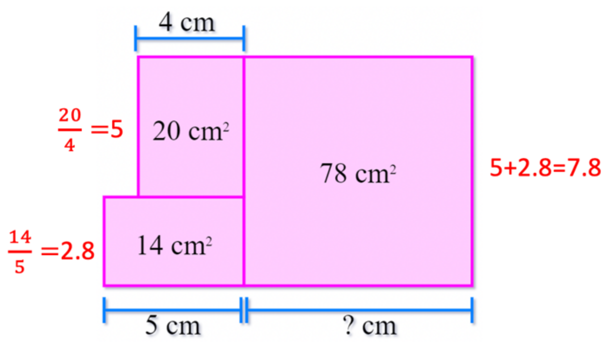
$11 - 1.5 - 1.5 = 8\ cm$

$8 - 1.5 - 1.5 = 5\ cm$

$8(5) = 40\ cm^2$

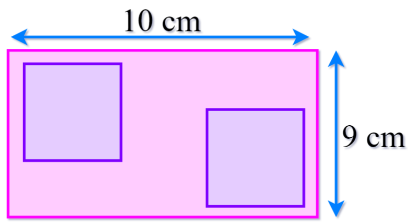
1.4 Working Backwards

18)



$$? = \frac{78}{7.8} = 10\ cm$$

19)



Area of rectangle$= 10(9) = 90$

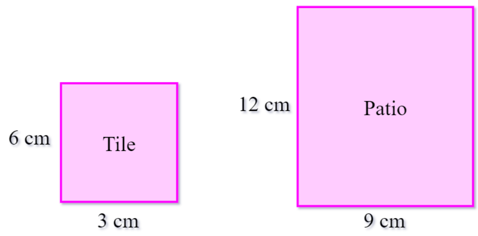
Area of 2 purple squares$= 90 - 58 = 32$

Area of 1 purple square$= \frac{32}{2} = 16$

Length of square $= \sqrt{16} = 4$ cm

1.5 Fitting

20)



Area of patio $=12(9) = 108\ cm^2$

Area of 1 tile$= 6(3) = 18\ cm^2$

$\frac{108}{18} = 6$ tiles

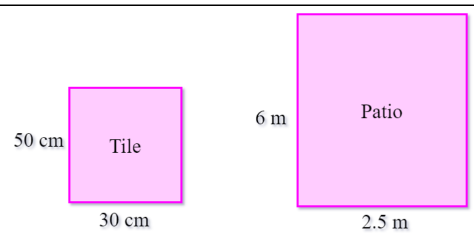
21)

Table area$= 55(60) = 300\ cm^2$

Sticker area$= 15(5) = 75\ cm^2$

$\frac{300}{75} = 4$ stickers

22)



Area of patio$= 6(2.5) = 15\ m^2$

Area of 1 tile$= 50(30) = 1500\ cm^2$

Match the measurements!

$100\ cm = 1\ m$

We need to square this since we have squared measurements

$10{,}000\ cm^2 = 1m^2$

Change all to cm	Change all to m
$15\ m^2 = 15(10000) = 150{,}000\ cm^2$ (area of patio) $\frac{150{,}000}{1500} =100$ tiles	$1500\ cm^2 = \frac{1500}{10{,}000} = 0.15\ m^2$ $\frac{15}{0.15}$ Kill the decimal $= \frac{1500}{15} = \frac{300}{3} = 100$

23)

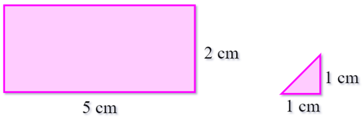
Area of floor$= 5(4) = 20\ m^2$

Area of 1 tile$= 50(50) = 2500\ cm^2$

Match the measurements!
100 cm =1 m
We need to square this (both sides) since we have squared measurements
$10{,}000\ cm^2 = 1m^2$

Change all to cm	Change all to m
$20\ m^2 = 20(10000) = 200{,}000\ cm^2$ (area of patio) $\frac{200{,}000}{2500} = 80$ tiles	$2000\ cm^2 = \frac{2500}{10{,}000} = 0.25\ m^2$ $\frac{20}{0.25}$ Kill the decimal $= \frac{2000}{25} = \frac{400}{5} = 80$

24)



i.

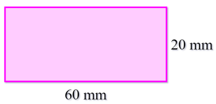
Area of rectangle $= 2(5) = 10\ cm^2$

ii.

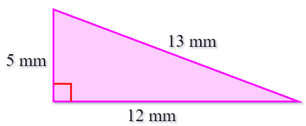
Area of triangle $= \frac{1}{2}(1)(1) = \frac{1}{2}$

$\frac{10}{\frac{1}{2}} = 20$ triangles

25)



George cuts the rectangle up into an exact number of right-angled triangles, each with sides as shown



i. Calculate the number of triangles that he cuts from the rectangle
ii. Find the combined perimeter of all the triangles that have been cut from the rectangle
iii. Convert this distance from millimetres into metres

i.

Area of rectangle $= 60(20) = 1200\ mm^2$

Area of triangle $= \frac{1}{2}(5)(12) = 30\ mm^2$

$\frac{1200}{30} = 40$ triangles

ii.

Perimeter of 1 triangle $= 5 + 13 + 12 = 30$

Perimeter of 40 triangles $= 40(30) = 1200\ mm$

iii.

$1200\ mm$

We need to change this into m

$$\frac{1200}{1000} = 1.2\ m$$

26) 1 m by 4 m rolls of turf cost £80.00. Mr Taylor's yard is 5 m long and 8 m wide. How much will it cost him to turf half of his yard?

Yard area $= 5(8) = 40\ cm^2$

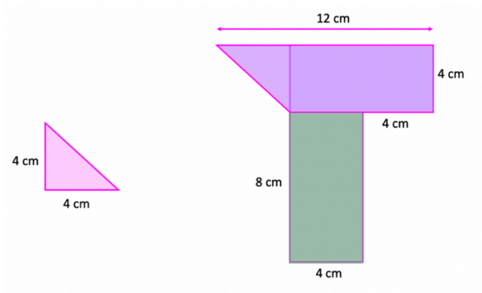
Turf area $= 1(4) = 4\ cm^2$

We want to turf half the yard which is $20\ cm^2$

$\frac{20}{4} = 5$ rolls

$$5(80) = £400$$

27)

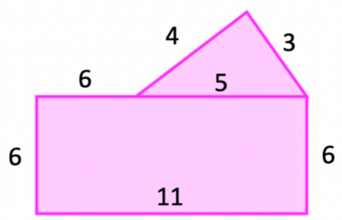


Total area = trapezium + rectangle $= \frac{1}{2}(12+8)(4) + 8(4) = 72$

Area of triangle $= \frac{4(4)}{2} = 8$

$\frac{72}{8} = 9$ tiles

28)



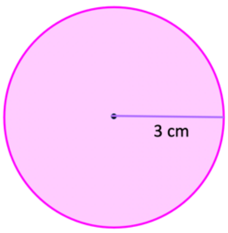
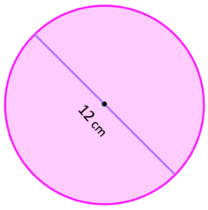
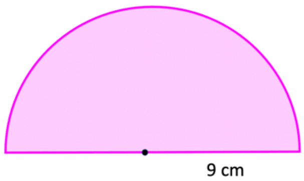
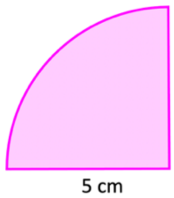
$$4 + 3 + 6 + 11 + 6 + 6 = 36\ cm$$

Note: we ignore the 5 since it is inside the shape, it is not part of the perimeter

2 Silver

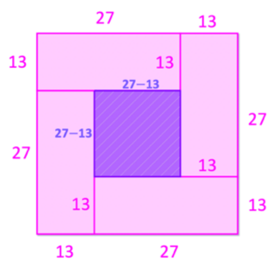
2.1 Simple Shapes

29) Find the area and perimeter of the shapes below

Circumference of circle $= 2\pi r$ Areae of circle $= \pi r^2$			
 `Circumference= $2\pi r$ $= 2\pi(3)$ $= 18.8\ cm$ Area= $\pi r^2 = \pi(3^2) = 28.3\ cm^2$	 Circumference $= 2\pi r$ $= 2\pi(6)$ $= 37.7\ cm$ Area= πr^2 $= \pi(6^2)$ $= 113.1 = cm^2$	 Perimeter= $\frac{2\pi(9)}{2} + 18$ $= 46.3\ cm$ Area= $\frac{\pi(9)^2}{2}$ $= 127.2\ cm^2$	 Perimeter= $\frac{2\pi(5)}{4} + 5 + 5$ $= 17.9\ cm$ Area= $\frac{\pi r^2}{4}$ $= \frac{\pi(5)^2}{4}$ $= 19.6\ cm^2$

2.2 Compound Shapes

30)



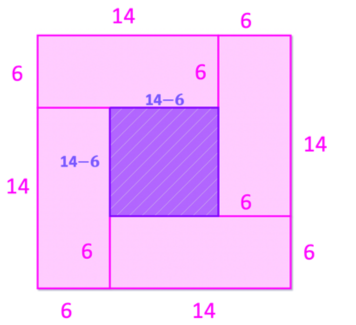
i.

Area of big square=40× $4 = 160\ cm^2$

ii.

$14 \times 14 = 186\ cm^2$

31)



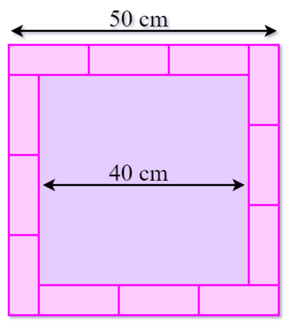
i.

Area of big square $=20\times 4 = 80\ cm^2$

ii.

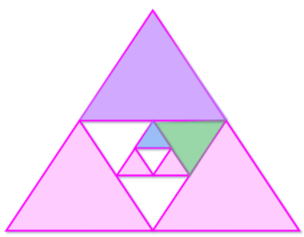
$8 \times 8 = 64\ cm^2$

32)



Way 1:	Way 2:
Width of each pink rectangle $= \frac{50-40}{2} = \frac{10}{2} = 5$ Length of each pink rectangle $= \frac{50-5}{3} = 15$ Area of pink rectangle = 5 × 15 $cm^2 = 75\ cm^2$	Find the area of the whole pink region: $50(50) - 40(40) = 900$ There are 12 rectangles Area of 1 pink rectangle $= \frac{900}{12} = 75\ cm^2$

33)

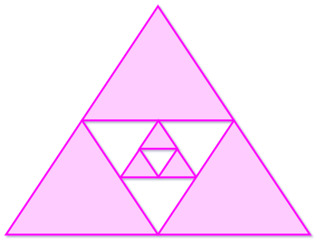


We can see that 4 large triangles fit in the shape, 4 medium triangles fit in a large triangle and 4 small triangles fit in a medium triangle

Area of large triangle $= \frac{128}{4} = 32$

Area of medium triangle$= \frac{32}{4} = 8$

Area of small triangle $= \frac{8}{4} = 2$

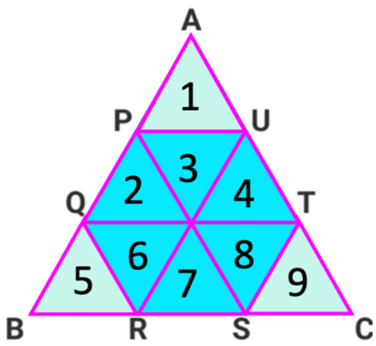


Total pink shaded area $=$3 large triangles $+$ 3 small triangles

$3(32) + 3(2) = 96 + 6 = 102\ cm^2$

34)

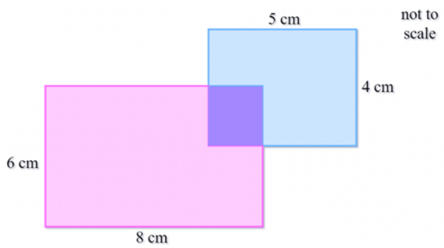
Hexagon is made up of 6 triangles



9 triangles total (6 blue and 3 green)

$\frac{90}{9} = 10$

35)



purple region $+$ blue shaded region $= 20$

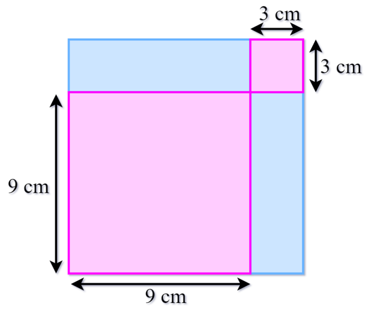
Given that area of blue shaded region $= 15$

purple region $+ 15 = 20$

purple region $= 5$

pink shaded region = area of full pink rectangle $-$ purple region $=(8)(6) - 5 = 48 - 5 = 42\ cm^2$

36)

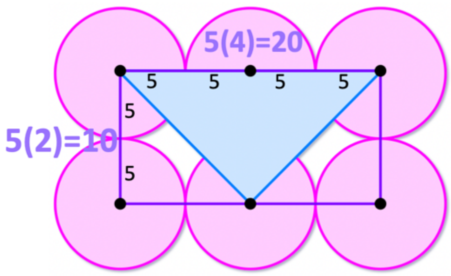


Total pink area= 9(9)+3(3)=90

Total blue area= 9(3)+9(3)=54

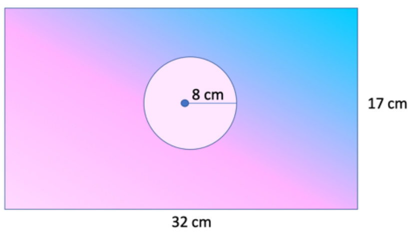
$90:54 = 10:6 = 5:3$

37)



Area $= 20(10) = 200\ cm^2$

38)

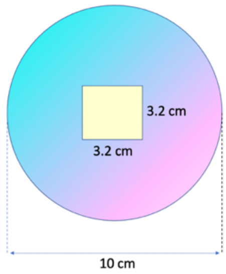


Area of rectangle= $32(17) = 544$

Area of circle $= \pi r^2 = \pi(8)^2 = 201.1$

Area of light pink shaded region $= 544 - 201.1 = 298.9\ cm^2$

39)

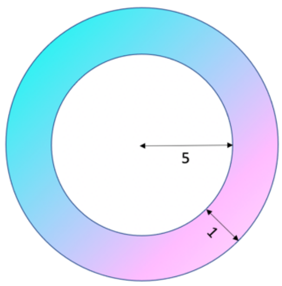


Area of full circle $= \pi r^2 = \pi(5)^2 = 78.5$

Area of yellow square$= 3.2(3.2) = 10.24$

Area of multi-coloured shaded region$= 78.5 - 10.24 = 68.3\ cm^2$

40)

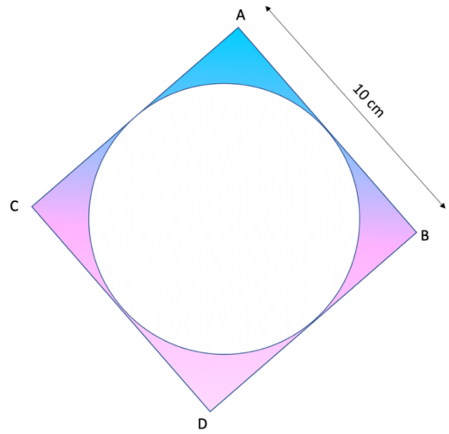


Area of larger circle $= \pi r^2 = \pi(6)^2 = 113.098$

Area of smaller circle$= \pi r^2 = \pi(5)^2 = 78.540$

Area of multi-coloured shaded region$= 113.098 - 78.540 = 34.6\ cm^2$

41)

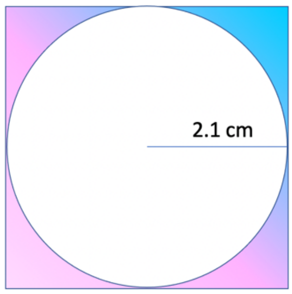


Area of square $= 10(10) = 100$

Area of circle$= \pi r^2 = \pi(5)^2 = 25\pi$

Area of multi-coloured shaded region$= 100 - 25\pi \ cm^2$

42)

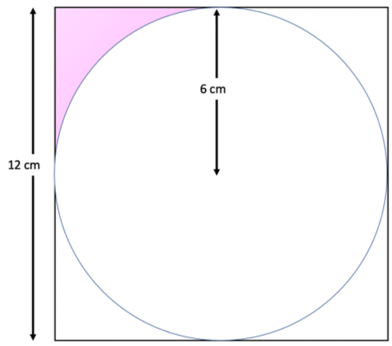


Area of square $= 4.2(4.2) = 17.64$

Area of circle$= \pi r^2 = \pi(2.1)^2 = 13.854$

Area of multi-coloured shaded region$= 17.64 - 13.854 = 3.79 \ cm^2$

43)



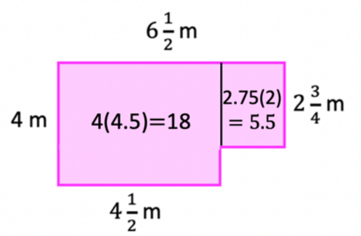
Area of square $= 12(12) = 144$

Area of circle$= \pi r^2 = \pi(6)^2 = 36\pi$

Area of pink shaded region$= \frac{144-36\pi}{4} = 36 - 9\pi \ cm^2$

2.3 Area Fitting

44)



i.

$$18 + 5.5 = 23.5\ m^2$$

ii.

area of tile $50(50) = 2500\ cm^2$

Match units!

$$100\ cm = 1\ m$$
$$10000cm^2 = 1m^2$$

$$23.5\ m^2 = 235{,}000\ cm^2$$

$$\frac{235{,}000}{2500} = 94$$

94 tiles

45)

Area of floor $= 3(2) = 6\ m^2$

Acorn tiles: 50 cm x 50 cm cost £4 each

$$50(50) = 2500cm^2$$

$$100\ cm = 1\ m$$
$$10000cm^2 = 1m^2$$

$$6\ m^2 = 60000\ cm^2$$

$$\frac{60000}{2500} = 24$$

$$24(£4) = £96$$

Beeching tiles: 60 cm x 40 cm. Cost £3 each

$$60(40) = 2400cm^2$$

$$\frac{60000}{2400} = 25$$

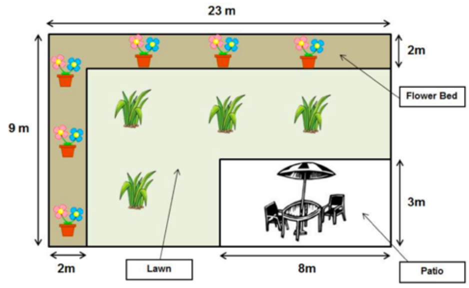
$$25(£3) = £75$$

Carpet: £14 per square metre. Fitting cost £30

$$6(14) + 30 = £114$$

Beeching Tiles is the cheapest

46)



i.

Area of lawn (including patio)= $21(7) = 147\ m^2$

Area of patio $8(3) = 24\ m^2$

Area of lawn $147 - 24 = 123\ m^2$

ii.

Total cost $4(6) = 24$
$10\%\ of\ 24 = 2.40$
$5\%\ of\ 24 = 1.20$
$15\%\ of\ 24 = 2.40 + 1.20 = £3.60$

$24 - 3.60 = £20.40$

iii.

patio is 8 m by 3m = 800 cm by 300 cm
$800 \div 50 = 16$
$300 \div 20 = 15$

Total number of tiles $15(16) = 240$

iv.

60 ÷ 4 = 15 sacks

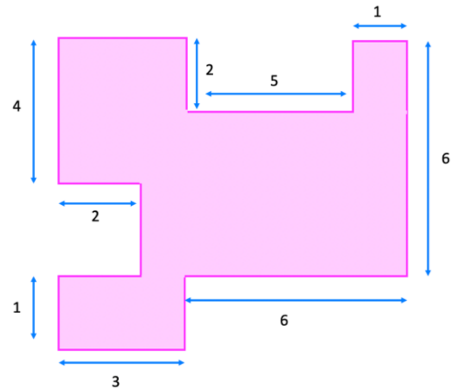
You pay for 12 sacks, and get 3 free

12 x 10 = £120

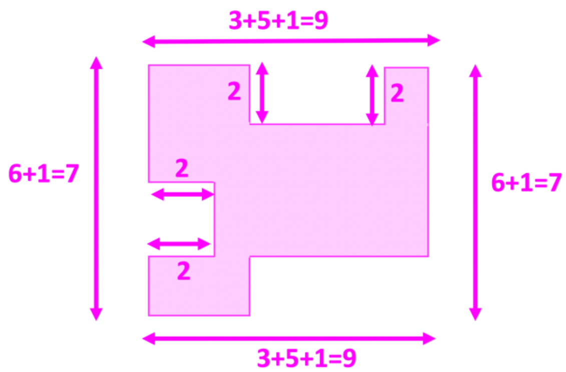
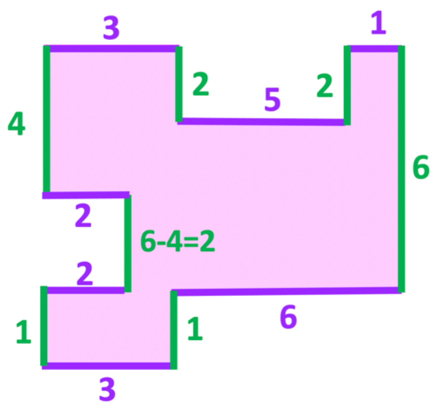
3 Gold

3.1 Compound Shapes

47) Find the area of the following shapes.

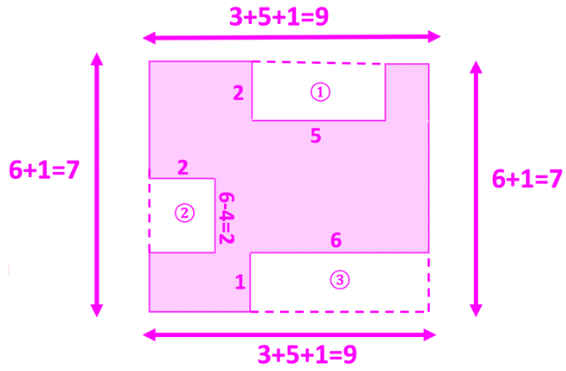


Perimeter

Way 1: Make into a complete rectangle	Way 2: Break each individual side down
	
$9+9+7+7+2+2+2+2=40\ cm$	$3+5+1+2+2+3+6+4+1+2+2+1+2+6=40$ $=cm$

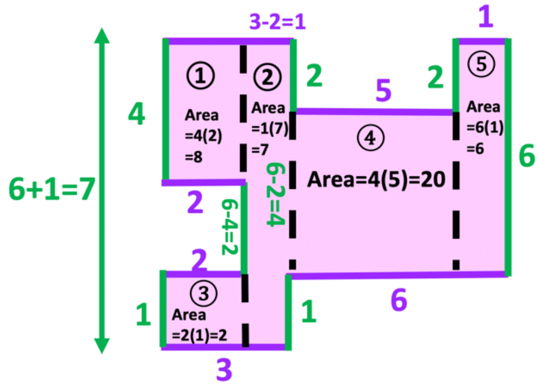
Area

Way 1: Make into a complete rectangle

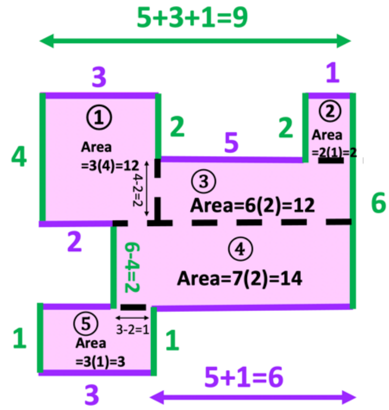


$63 - 2(5) - 2(2) - 1(6) = 43\ cm^2$

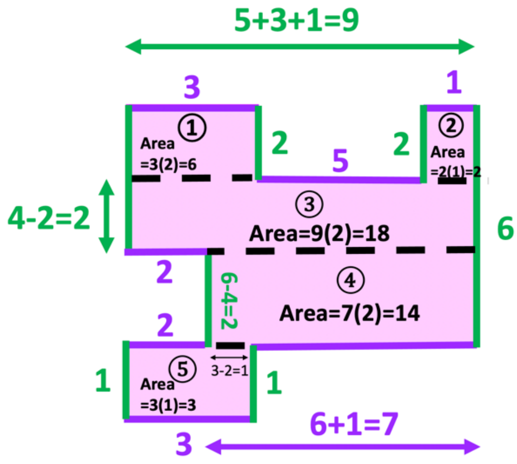
Way 2: Break each individual side down



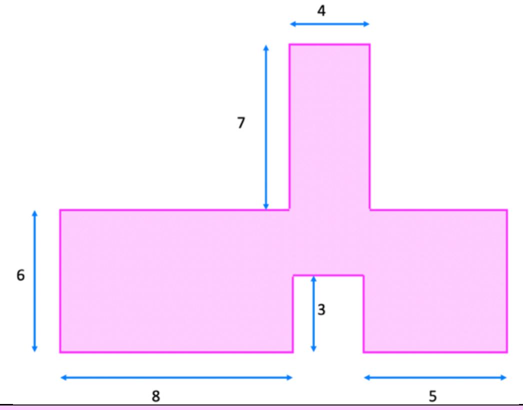
Area $= 8 + 7 + 2 + 20 + 6 = 43\ cm^2$



Area= 12+2+12+14+3=43 cm^2

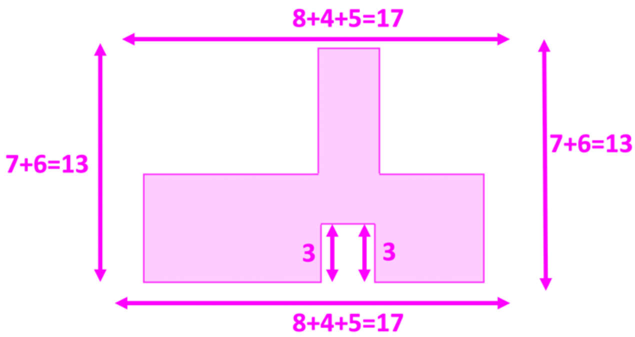


Area=6+2+18+14+3=43 cm^2



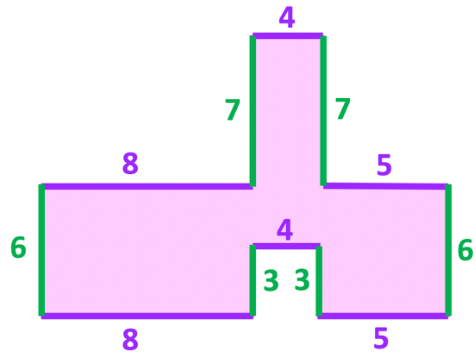
Perimeter

Way 1: Make into a complete rectangle



$13 + 13 + 17 + 17 + 3 + 3 = 66\ cm$

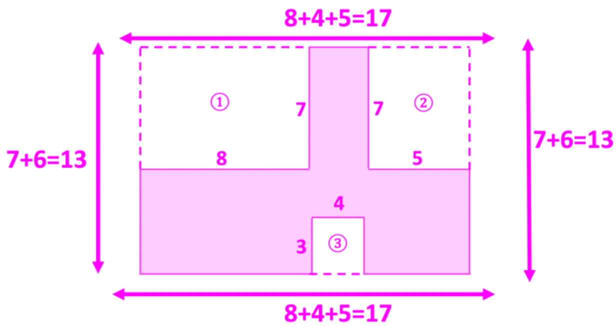
Way 2: Break each individual side down



$8 + 4 + 5 + 8 + 5 + 4 + 6 + 6 + 7 + 7 + 3 + 3 = 66\ cm$

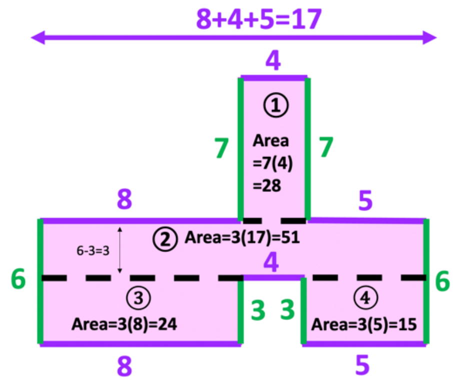
Area

Way 1: Make into a complete rectangle

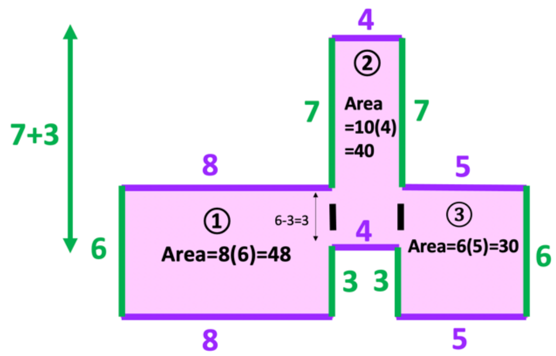


$17(13) - 8(7) - 7(5) - 3(4) = 118\ cm^2$

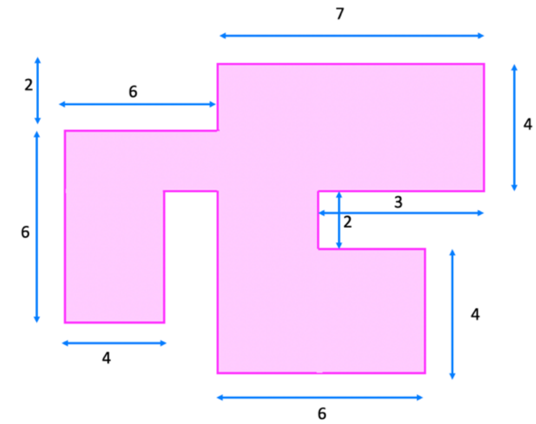
Way 2: Break each individual side down



$28 + 51 + 24 + 15 = 118\ cm^2$

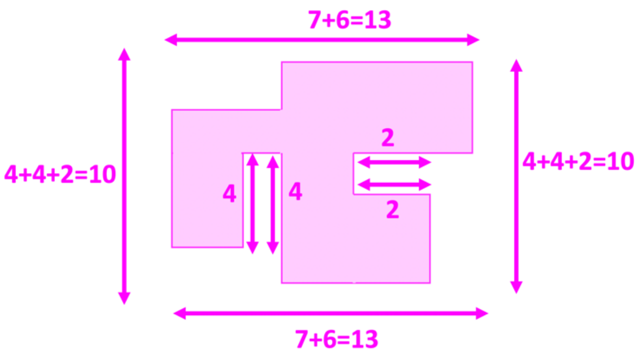


$48 + 40 + 30 = 118\ cm^2$



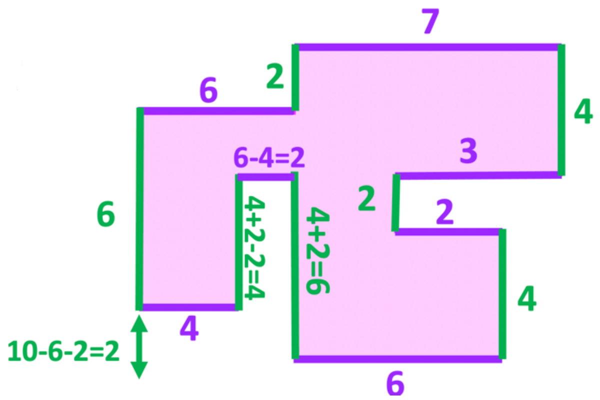
Perimeter

Way 1: Make into a complete rectangle



$13 + 13 + 10 + 10 + 4 + 4 + 2 + 2 = 58\ cm$

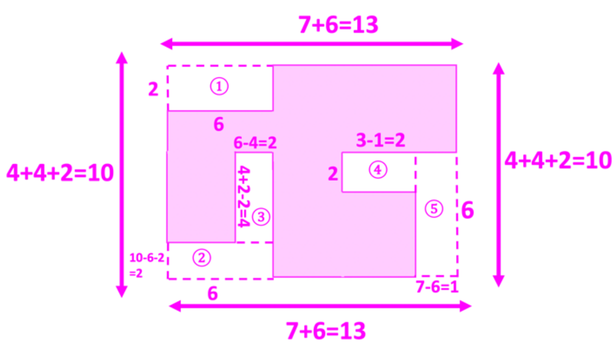
Way 2: Break each individual side down



$6 + 7 + 2 + 3 + 4 + 6 + 2 + 6 + 2 + 4 + 6 + 2 + 4 + 4 = 58\ cm$

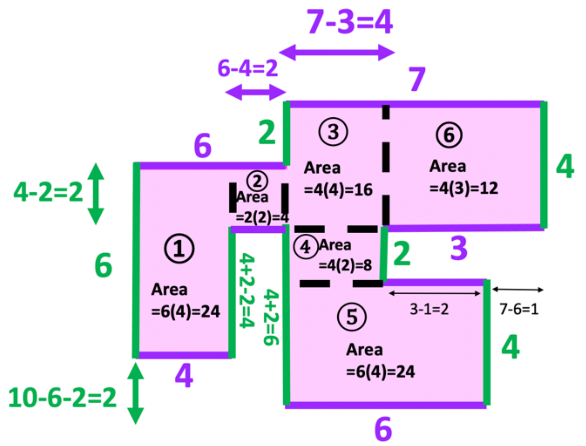
Area

Way 1: Make into a complete rectangle

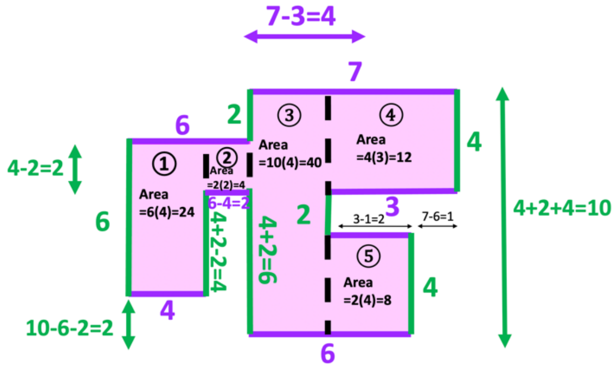


$13(10) - 2(6) - 2(6) - 2(4) - 2(2) - 6(1) = 88\ cm^2$

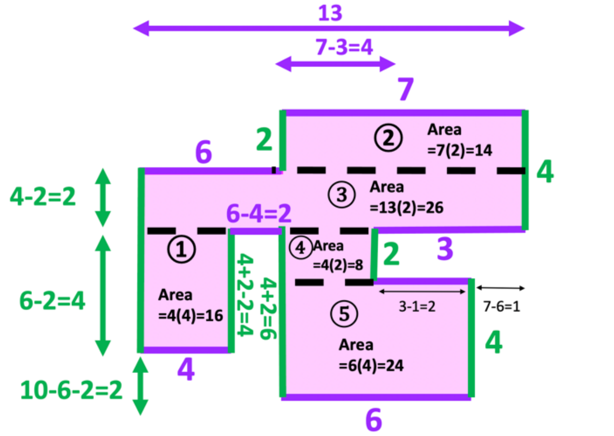
Way 2: Break each individual side down



$24 + 4 + 16 + 8 + 24 + 12 = 88\ cm^2$

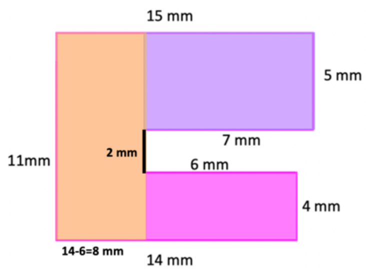


$24 + 4 + 40 + 12 + 8 = 88\ cm^2$



$16 + 14 + 26 + 8 + 24 = 88\ cm^2$

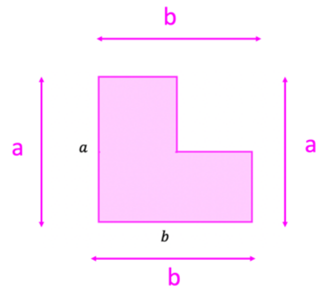
48)



Perimeter$= 15 + 5 + 7 + 2 + 6 + 4 + 14 + 11 = 64\ mm$

Area $= 7(5) + 6(4) + 11(8) = 35 + 24 + 88 = 147\ mm^2$

49)

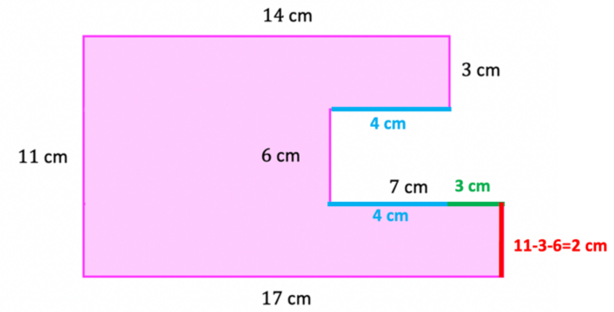


This looks like we don't know all side lengths, but it doesn't matter

$$a + a + b + b = 2a + 2b$$

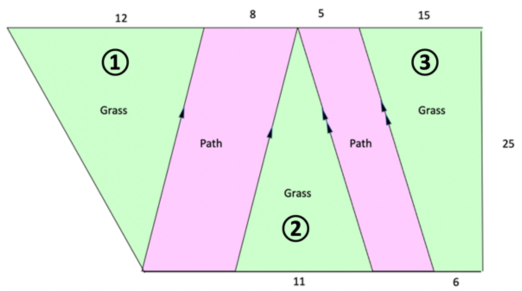
a) $2a + 2b$
b) $a + b$
c) $a \times b$
d) $2a \times 2b$
e) $2a + b - ab$

50)



perimeter$= 14 + 3 + 4 + 6 + 7 + 2 + 17 + 11 = 64\ cm$

51)



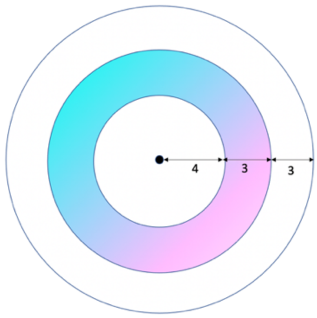
Area of ① : $\frac{12 \times 25}{2} = 150\ ft^2$

Area of ②: $\frac{11 \times 25}{2} = 137.5\ ft^2$

Area of ③: $\frac{25(15+6)}{2} = 262.5\ ft^2$

Total grass area $150 + 137.5 + 262.5 = 550\ ft^2$

52)



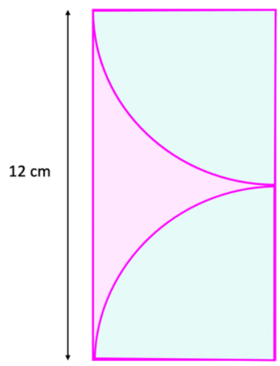
Area of small circle : $\pi r^2 = \pi(4)^2 = 16\pi$
Area of medium circle : $\pi r^2 = \pi(7)^2 = 49\pi$
Area of large circle : $\pi r^2 = \pi(10)^2 = 100\pi$

Shaded area = area of medium circle – area of small circle$= 49\pi - 16\pi = 33\pi$

We now want the fraction that is shaded: $\frac{shaded}{total} = \frac{33\pi}{100\pi} = \frac{33}{100}$

$\frac{33}{100} \neq \frac{1}{3}$, so Daisy is wrong

53)



Area of rectangle$= 12(6) = 72$

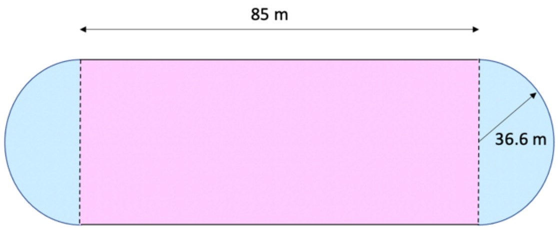
Area of a full circle$= \pi(6^2) = 36\pi$

We have two quarter circles which make a semicircle

Area of semi-circle$= \pi(6^2) = \frac{36\pi}{2} = 18\pi$

$72 - 18\pi = 15.5\ cm^2$

54)

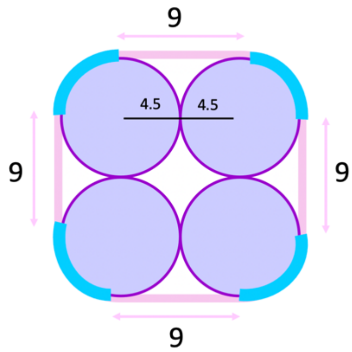


Area of rectangle$= 85(73.2) = 6222$

The blue semi-circles make a complete circle $= \pi r^2 = \pi(36.6)^2 = 4208.351$

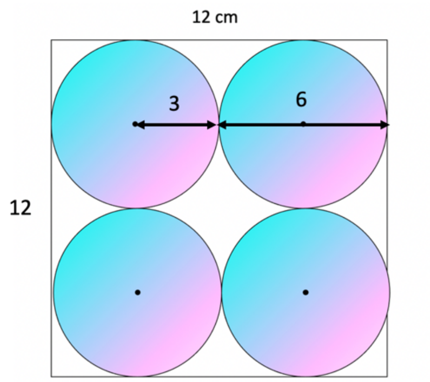
$4208.351 + 6222 = 10430.35 = 1400\ cm^2$ to 2sf

55)



The 4 curved edges make 1 full circle: $2\pi(4.5) = 9\pi$
Length$= 9(4) + 9\pi = 36 + 9\pi = 64.3$ mm

56)



Area of square: $(12)(12) = 144$

The circles each have radius 3

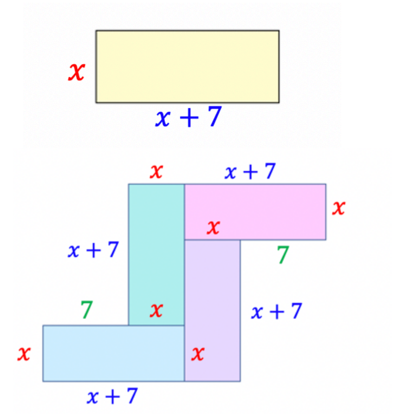
Area of 1 circle: $\pi(3)^2 = 9\pi$
Area of 4 circles : $4(9\pi) = 36\pi$

Area of unshaded region $144 - 36\pi = 36(4 - \pi)\ cm^2$

4 Diamond

4.1 Compound Shapes

57)



Perimeter $4x + 4(x + 7) + 14 = 8x + 42$

$$8x + 42 = 70$$
$$8x = 28$$
$$x = 3.5$$

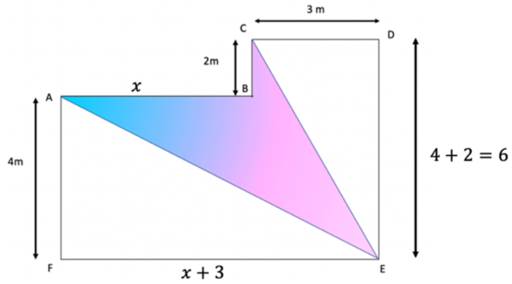
The width of the rectangle is $x = 3.5$
The length of the rectangle is $3.5 + 7 = 10.5$

Area of the rectangle $10.5(3.5) = 36.75$

The shape is made out of 4 rectangles

$4(36.75) = 147\ cm^2$

58)



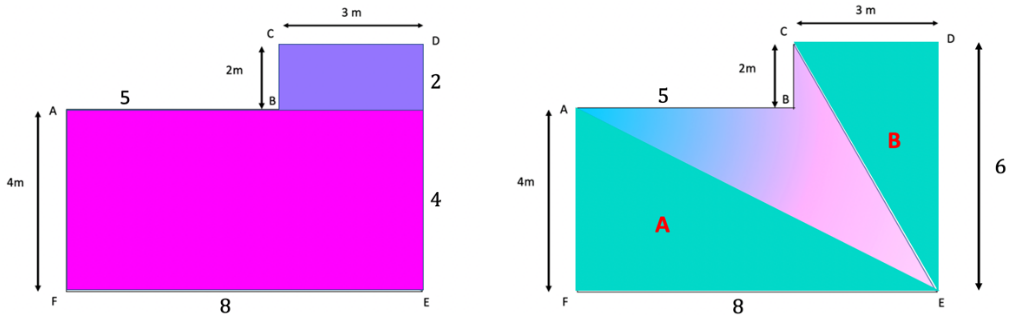
We know that the perimeter is 28

$$2+3+6+x+3+4+x=28$$
$$2x+20=30$$
$$2x=10$$
$$x=5$$

EF$= x+3=5+3=8$

Area of the multi-coloured shaded region = area of L shape −area of 2 triangles

so, we need the area of L shape and the 2 triangles



Area of L shape$= 2(3)+4(8)=6+32=38$

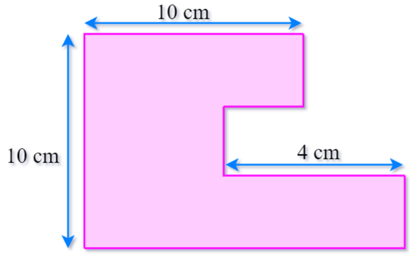
Area of A$= \frac{1}{2}(3)(6)=9$

Area of B $= \frac{1}{2}(4)(8)=16$

Area of the multi-coloured shaded region = area of L shape −area of 2 triangles

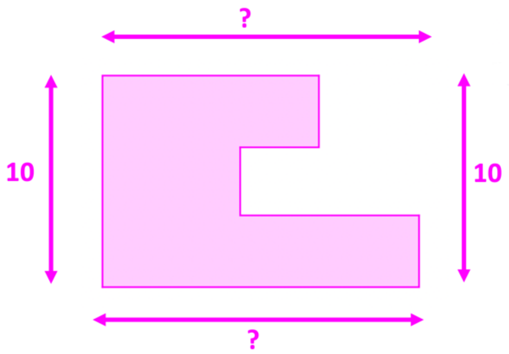
Area of multi-coloured shaded region$= 38-9-16=13\ m^2$

59)



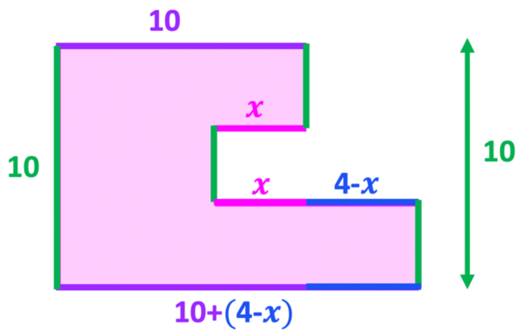
Perimeter

Way 1: Make into a complete rectangle



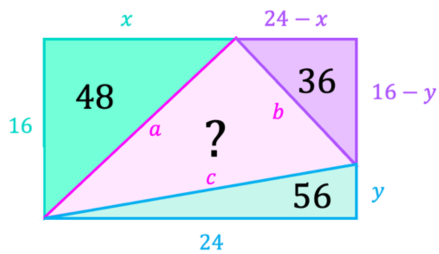
We can't do it this way as we don't know the ? lengths

Way 2: Break each individual side down



$10 + 10 + 10 + x + x + (4 - x) + 10 + (4 - x) = 40 + 8 = 48\ cm$

60)



$$a + 16 + x = 48$$

$$b + 24 - x + 16 - y = 36$$

$$y + 24 + c = 56$$

Add all 3 lines together

$$a + 16 + b + 24 + 16 + 24 + c = 48 + 36 + 56$$

Simplify and you will see an expression for $a + b + c$ 'pops out'

$$a + b + c + 80 = 140$$

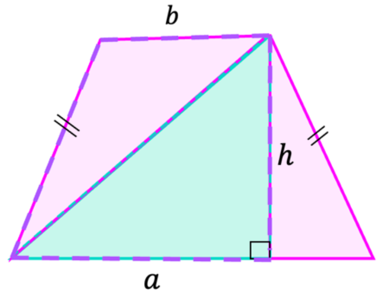
$$a + b + c = 60$$

61)

Way 1:

Move the yellow triangle on the right to the left triangle such that it forms another right triangle with it. The base and height are the same as the orange therefore the same

Way 2:



Green area $= \frac{1}{2}(a)(h)$

Top yellow triangle area $=$ trapezium $-$ green triangle $= \frac{1}{2}(a+b)h - \frac{1}{2}(a)(h)$

Bottom yellow triangle $= \frac{1}{2}(a-b)h$

Total yellow area = top yellow area + right yellow area $= \frac{1}{2}(a+b)h - \frac{1}{2}(a)(h) + \frac{1}{2}(a-b)h = \frac{1}{2}ah$

Therefore, the same

62)

This can be done without algebra

Yellow + pink = triangle half the area of the square

In order to stretch the pink triangle to become the above triangle half the size of the square, we stretch the base by 3 times and the height by 4 times (triangle opposite pink has 3 times the height due to similar triangles)

Thus,

pink area is $\frac{1}{12}$ of the triangle

Yellow area is $\frac{11}{12}$ of the triangle

$$\frac{11}{12} \times 6 = 5.5$$

Alternative method: With algebra

The pink triangle is similar to the bigger blue triangle hence the lengths will be in ratio and the area scale factor can be calculated etc

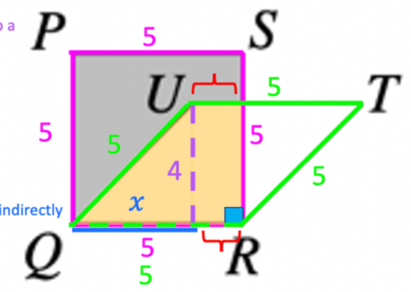
5 Challenges

63)

All the pink lengths are 5 since the area is 25 and we have a square where all side lengths are the same

One of the sides of a rhombus is the side of the square hence we know the rhombus has all side lengths 5 since all sides of a rhombus have the same length

The rhombus has area 20 which is base times height hence the height is 4 (we can drop a perpendicular down to represent the height)



First question: What does the shaded region consist of?
The shaded region consists of a square minus a trapezium

We already know the area of the square already

Second question: Can I find the trapezium area directly?
Yes if we knew the top parallel side length. We can use Pythagoras to help us find this indirectly

$$x^2 + 4 = 5^2$$
$$x^2 = 9$$
$$x = 3$$

Now we can find the red length

$$5 - 3 = 2$$

Trapezium area $= \frac{1}{2}(\text{sum of parallel sides})\times\text{height} = \frac{1}{2}(2+5)\times 4 = 14$

Shaded region = area of square minus trapezium $= 25 - 14 = 11\ cm^2$

$11\ cm^2$

64)

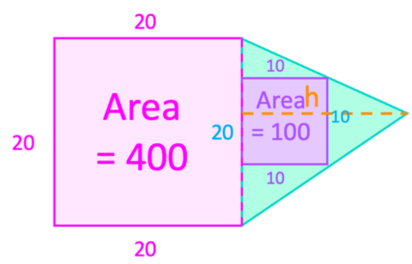
The combined green and pink areas $= 11(11) + 7(7) = 170$

The combined pink and purple areas $= 9(9) + 5(5) = 106$

Since the pink area is included in both, the difference between the total will be the difference between green and purple

$$170 - 106 = 64$$

65)



Triangle BCD and triangle ACE are similar, hence all sides are multiplied by a scale factor

Scale factor $= \frac{20}{10} = 2$

This means ratio 2:1

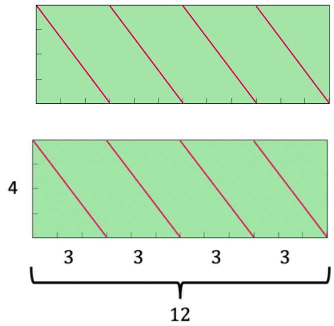
Height $= 2(10) = 20$

Area of triangle ACE $= \frac{1}{2}(20)(20) = 200$

$$200 - 100 = 100$$

66)

We are going to cut the tube, or imagine cutting the tube, at the point where the rope first appears and lay it flat.



$$3^2 + 4^2 = x^2$$

$$x^2 = 25$$

$$x = 5$$

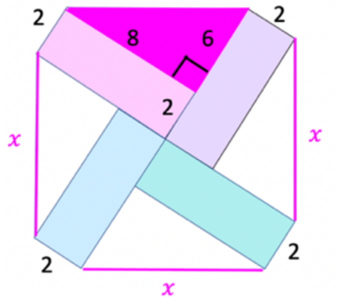
4 diagonals

$$4(5) = 20$$

Note: There is no need to double since the rectangle is the entire cylinder unfolded

67)

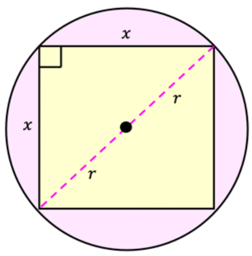
$$x = \sqrt{8^2 + 6^2}$$
$$= \sqrt{100} = 10$$



$$x^2 = 8^2 + 6^2$$
$$x^2 = 100$$
$$x = 10$$

Perimeter $4(10) + 4(2) = 48$ cm

68)

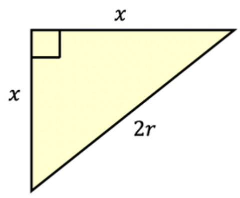


Let's first use the fact that we know the area of the circle

$$\pi r^2 = 49$$

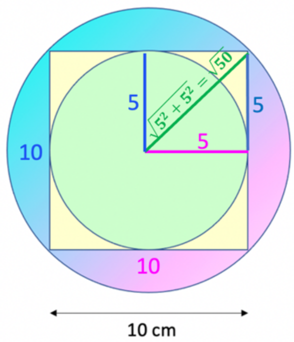
$$r^2 = \frac{49}{\pi}$$
$$r = \sqrt{\frac{49}{\pi}}$$
$$r = \frac{7}{\sqrt{\pi}}$$

Now we can use Pythagoras



$$x^2 + x^2 = \left(2 \times \frac{7}{\sqrt{\pi}}\right)^2$$
$$x^2 + x^2 = \left(\frac{14}{\sqrt{\pi}}\right)^2$$
$$2x^2 = \frac{196}{\pi}$$
$$x^2 = \frac{98}{\pi}$$
$$x = \sqrt{\frac{98}{\pi}}$$
$x = 5.59$ cm

69)



Radius of inner green circle $= \frac{10}{2}$

Area of inner green circle $= \pi r^2 = \pi \times 5^2 = 25\pi$

Radius of outer circle = distance from centre to corner of square

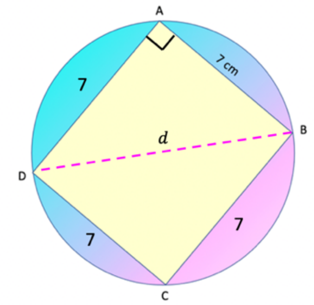
Using Pythagoras: $r^2 = 5^2 + 5^2 = 50 \Longrightarrow r = \sqrt{50}$

Area of outer circle $= \pi r^2 = \pi \times 50 = 50\pi$

Shaded area between the two circles$= 50\pi - 25\pi = 25\pi$

$\therefore$ shaded area between the two circles = area of inner green circle$= 25\pi\ cm^2$

70)



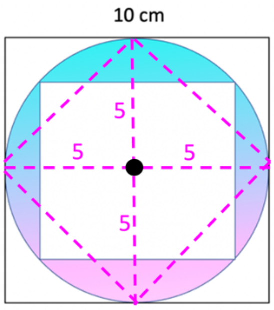
$$7^2 + 7^2 = d^2$$
$$d = \sqrt{98}$$
Area of circle $= \pi r^2 = \pi\left(\frac{\sqrt{98}}{2}\right)^2 = \frac{49}{2}\pi$

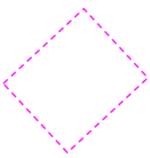
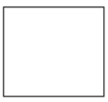
Area of square $7 \times 7 = 49$

Area of multicoloured shaded region$= \frac{49}{2}\pi - 49 = 27.969 = 28\ cm^2$

71)

Way 1:

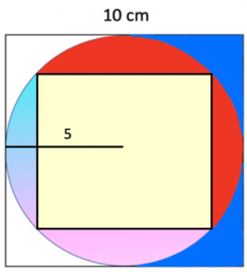


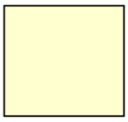
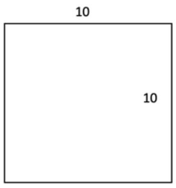
 =  = 4 triangles = $4\left[\frac{1}{2}(5)(5)\right]$

Circle area 25π

Shaded region area $= 25\pi - 50 = 28.5\ cm^2$

Way 2:



 = 

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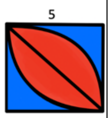
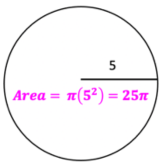
5 − 

Shaded region =



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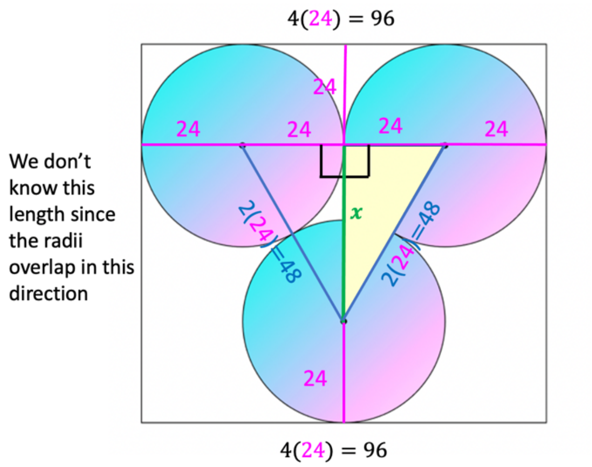
$= 25\pi - 50 = 28.5$

72)

Way 1

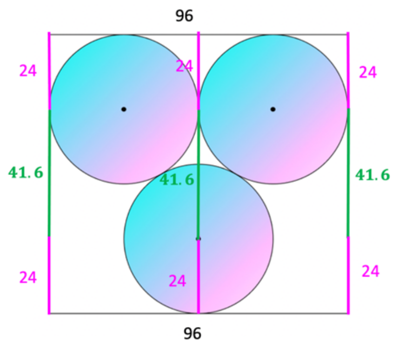
We know that the length of the top and bottom sides are the same as 4 of the radii lengths hence $4(24) = 96$



Use Pythagoras on the yellow triangle:

$$x^2 + 24^2 = 48^2$$
$$x^2 = 48^2 - 24^2 = 1728$$
$$x = 41.6$$

So now we can find the length of the side we didn't know

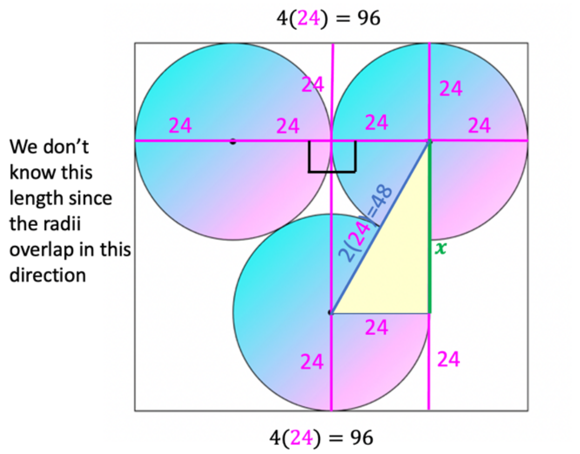


$24 + 24 + 41.6 = 89.569$

Area = length ×height= $96(89.569) = 8598.624 = 8600\ mm^2$

Way 2

We know that the length of the top and bottom sides are the same as 4 of the radii lengths hence $4(24) = 96$



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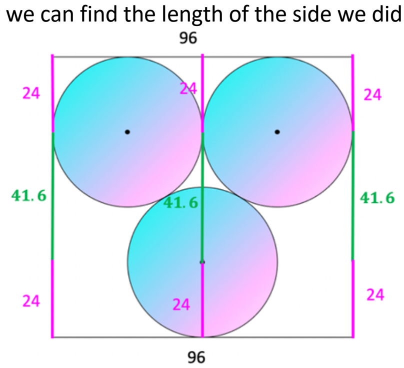
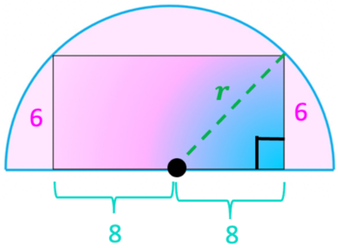


$24 + 24 + 41.6 = 89.569$

Area = length ×height= $96(89.569) = 8598.624 = 8600$ mm^2

73)



$r^2 = 6^2 + 8^2$

$r^2 = 36 + 64$

$r^2 = 100$

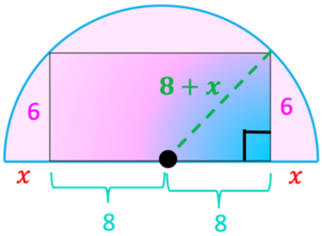
$r = 10$

Area of semicircle $= \frac{\pi r^2}{2} = \frac{\pi(10)^2}{2} = 50\pi$

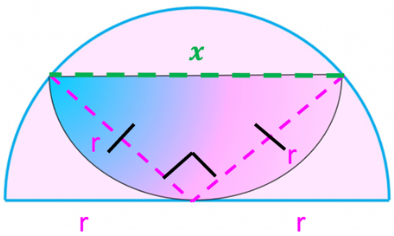
Area of multicoloured rectangle $16(6) = 96$

Light pink shaded area $(50\pi - 96)\ cm^2$

Note: We could have labelled the question as the following, but it would have produced a quadratic to solve

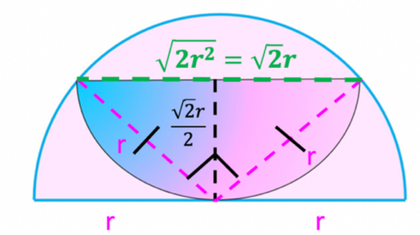


74)



We need to use Pythagoras to find the green length

$r^2 + r^2 = x^2$
$2r^2 = x^2$
$x^2 = 2r^2$
$x = \sqrt{2r^2}$
$x = \sqrt{2}r$



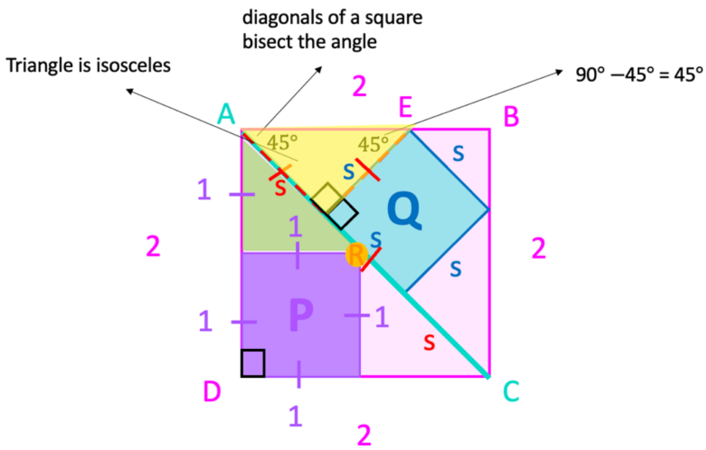
Area of large semicircle: $\frac{\pi r^2}{2} = \frac{1}{2}\pi r^2$

Area of small semicircle: $\frac{\pi\left(\frac{\sqrt{2}}{2}r\right)^2}{2} = \frac{\pi\left(\frac{1}{2}r^2\right)}{2} = \frac{1}{4}\pi r^2$

$$\frac{1}{4}\pi r^2 : \frac{1}{2}\pi r^2 = \frac{1}{4} : \frac{1}{2} = 1:2$$

So ratio of smaller semicircle to larger semicircle= 1: 2

75)



For convenience choose length of square to be ABCD to be 2 (doesn't matter what we pick as looking for the ratio)

Corner of R is the midpoint hence the triangles are isosceles and identical

Let's apply Pythagoras to triangle ABC

$$2^2 + 2^2 = AC^2$$
$$AC^2 = 8$$
$$AC = \sqrt{8} = 2\sqrt{2}$$

$$AR = \frac{1}{2}AC = \sqrt{2}$$

Let's now look at the yellow triangle and use Pythagoras

$$s^2 + s^2 = AE^2$$
$$2s^2 = AE^2$$
$$AC = \sqrt{2s^2} = \sqrt{2}s$$

Now we can use the fact that we length of the diagonal to help us find S

$$s + s + s = 2\sqrt{2}$$

$$3s = 2\sqrt{2}$$

$$s = \frac{2\sqrt{2}}{3}$$

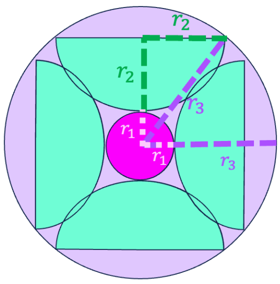
Area of Q $= s^2 = \left(\frac{2\sqrt{2}}{3}\right)^2 = \frac{8}{9}$

Area of P $= 1(1) = 1$

$$1 : \frac{8}{9}$$

$$9 : 8$$

76)



Using the fact that the area of the inner pink circle is 4:

$$\pi r_1^2 = 4$$

$$r_1^2 = \frac{4}{\pi}$$

$$r_1 = \sqrt{\frac{4}{\pi}}$$

$$r_1 = \frac{2}{\sqrt{\pi}}$$

Using the fact that the area of the semicircle circle is 18:

$$\frac{\pi r_2^2}{2} = 18$$

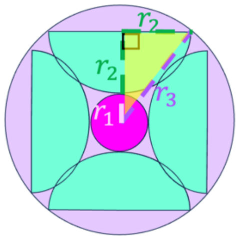
$$\pi r_2^2 = 36$$

$$r_2^2 = \frac{36}{\pi}$$

$$r_2 = \sqrt{\frac{36}{\pi}}$$

$$r_2 = \frac{6}{\sqrt{\pi}}$$

We can form a right-angled triangle with the radii and use Pythagoras:



$$r_2^2 + (r_1 + r_2)^2 = r_3^2$$

$$r_2^2 + r_1^2 + 2r_1r_2 + r_2^2 = r_3^2$$

$$\left(\frac{6}{\sqrt{\pi}}\right)^2 + \left(\frac{2}{\sqrt{\pi}}\right)^2 + 2\left(\frac{6}{\sqrt{\pi}}\right)\left(\frac{2}{\sqrt{\pi}}\right) + \left(\frac{6}{\sqrt{\pi}}\right)^2 = r_3^2$$

$$r_3^2 = \frac{36}{\pi} + \frac{4}{\pi} + \frac{24}{\pi} + \frac{36}{\pi}$$

$$r_3^2 = \frac{100}{\pi}$$

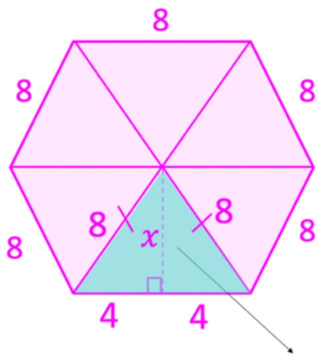
Area of outer circle:

$$\pi r_3^2$$

$$\pi\left(\frac{100}{\pi}\right)$$

$$= 100$$

77)



Firstly, we need to use Pythagoras to find x

$$x^2 + 4^2 = 8^2$$

$$x = 6.928$$

Area of triangle $= \frac{1}{2}(8)(6.928) = 27.712$

we have 6 triangles

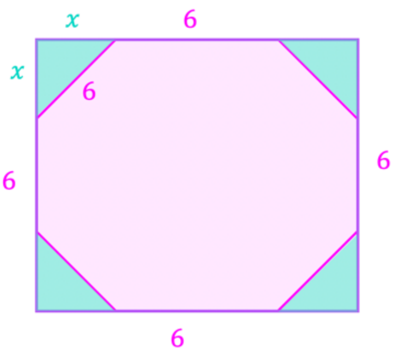
$6 \times 27.712 = 166.3\ units^2$

Equilateral triangle since $\frac{360}{6} = 60$

$166.3\ units^2$

78)

We can form the 4 triangles and then do the area of the square minus the area of the four triangles



Firstly, we need to use Pythagoras to find x

$$x^2 + x^2 = 6^2$$
$$2x^2 = 36$$
$$x^2 = 18$$
$$x = \sqrt{18}$$

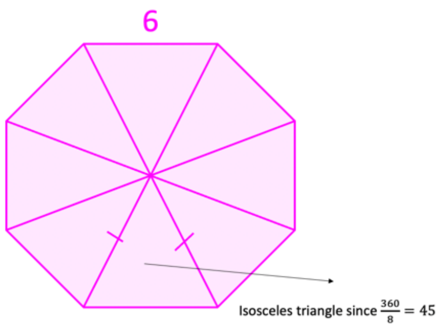
Area of 1 triangle$=\frac{1}{2}\sqrt{18}\,\sqrt{18}=9$

Area of square $= (6+2\sqrt{18})(6+2\sqrt{18}) = 209.8$

Square – 4 triangles=209.8-36 = 173.8 $units^2$

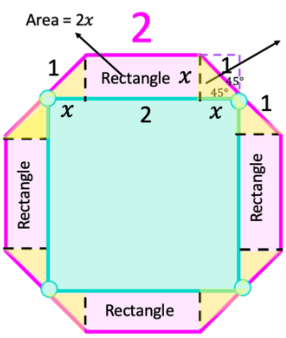
173.8 $units^2$

Note: If we tried to do this question like we did with hexagon where we split it into 8 triangles, we have a problem since the triangles are not equilateral. We could do it this way if we use some trigonometry (SOHCAHOTA), but you haven't come across this yet.



79)

Way 1: Area of 8 triangles + 4 rectangles



This triangle is isosceles because the angle between the square and octagon is 45. Dropping a perpendicular line, by definition, creates a 90 angle, so the third angle is 180 – (90+45) = 45

Using Pythagoras on yellow triangle to find x

$$x^2 + x^2 = 1$$
$$x^2 = \frac{1}{2}$$
$$x = \frac{1}{\sqrt{2}}$$

The red area consists of the area of 4 rectangles plus 8 triangles

Way 2: Find the area of 4 trapeziums (build squares in all 4 corners to help)

- Call a side length of the built in square x and use Pythagoras on the triangle to find x
- We know the height of the trapezium is $\frac{1}{2}x$
- We need to find the length of the square for the base length of the trapezium.

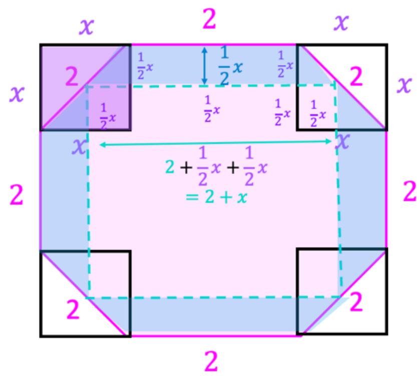
Area of 1 triangle = $\frac{1}{2}x^2 = \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{4}$

Area of 1 rectangle $= 2x = 2\left(\frac{1}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}}$

Area of 8 triangles $= 8\left(\frac{1}{4}\right) = 2$

Area of 4 rectangles $= 4\left(\frac{2}{\sqrt{2}}\right) = \frac{8}{\sqrt{2}} = 4\sqrt{2}$

$$2 + 4\sqrt{2} = 7.65$$



Each slanted side of the octagon can be viewed as the diagonal of a square, whose side is twice as long as the width of the blue area. That diagonal has length 2, so using Pythagoras that square has side $\sqrt{2}$

$$x = \sqrt{2}$$

The original blue trapezium has height $\frac{1}{2}x = \frac{\sqrt{2}}{2}$

The side of the original green square is $2 + \sqrt{2}$

Each blue trapezoid has an area of

$$\frac{1}{2}(2 + x)\times\text{height}$$

$$\frac{1}{2}(2 + 2 + \sqrt{2})\,\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}(4 + \sqrt{2}) = \sqrt{2} + \frac{1}{2}$$

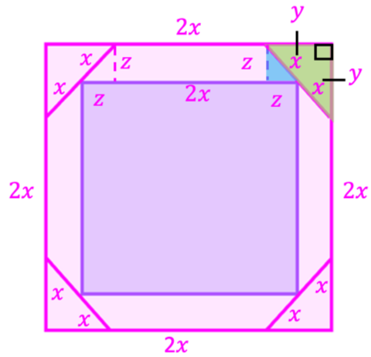
The TOTAL blue area is four times as big.

$$\boxed{4\sqrt{2} + 2}$$

Extension:
From here we can quickly figure out the area of the octagon: since the blue square has area $(2 + \sqrt{2})^2 = 6 + 4\sqrt{2}$
add the red area to get $8 + 8\sqrt{2}$.
In general, the area of a regular octagon of side length s is

$$2(1 + \sqrt{2})\,s^2$$

80)



Note: We could call the sides of the octagon x like usual, but since we will be halving them to work on the blue triangle it is easier to call them $2x$. In fact, to make the algebra the nicest (to avoid having to rationalize), we should have really called the sides $4x$

$$y^2 + y^2 = (2x)^2$$

$$2y^2 = 4x^2$$

$$y^2 = 2x^2$$

$$y = \sqrt{2}x$$

$$z^2 + z^2 = x^2$$

$$2z^2 = x^2$$

$$z^2 = \frac{x^2}{2}$$

$$z = \frac{x}{\sqrt{2}}$$

Length of square $= 2x + z + z = 2x + \frac{x}{\sqrt{2}} + \frac{x}{\sqrt{2}} = 2x + \frac{2x}{\sqrt{2}} = 2x + \sqrt{2}x = x(2 + \sqrt{2})$

Length of outer square $= 2x + y + y = 2x + \sqrt{2}x + \sqrt{2}x = 2x + 2\sqrt{2}x = x(2 + 2\sqrt{2})$

$$\sqrt{2}(2 + \sqrt{2}) = 2 + 2\sqrt{2}$$

The squares have sides in the ratio $1:\sqrt{2}$

Therefore the squares have areas in the ratio $1:2$